Óbuda University

Theses of Doctoral (PhD) Dissertation

Intelligent Decision Models

Author:
Orsolya Csiszár

Supervisors:
Prof. Dr. János Fodor
Prof. Dr. Róbert Fullér

Doctoral School of Applied Informatics and Applied Mathematics

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1. Background of the research: aggregation and decision

Aggregation is the process of combining several numerical values into a single representative one. The function, which performs this process is called an aggregation function. Despite the simplicity of this definition, the size of the field of its applications is incredibly huge: applied mathematics (e.g. probability theory, statistics, decision theory), computer sciences (e.g. artificial intelligence, operation research, pattern recognition and image processing), economics and finance, multicriteria decision aid, etc. (see e.g. Beliakov et al., 2007, Grabisch et al., 2009).

If we think of the arithmetic mean, we can see that the history of aggregation is as old as mathematics itself. However, it was only in the last decades, when the rapid development of the above mentioned fields (mainly due to the arrival of computers) made it necessary to establish a sound theoretic basis for aggregation. The problem of data fusion, synthesis of information or aggregating criteria to form overall decision is of considerable importance in many fields of human knowledge. Due to the fact that data is obtained in an easier way, this field is of increasing interest.

One of the most prominent group of applications of aggregation functions comes from decision theory. Making decisions often leads to aggregating preferences or scores on a given set of alternatives, the preferences being obtained from several decision makers, experts, voters or representing different points of view, criteria and objectives. This concerns decision under multiple criteria or multiple attributes, multiperson decision making and multiobjective optimization (Fodor and Roubens, 1994).

Another outstanding application of aggregation functions comes from artificial intelligence, fuzzy logic (Dubois et al., 2000). Pattern recognition and classification, as well as image analysis are typical examples. According to Aristotle, in mathematics it was originally assumed that "the same thing cannot at the same time both belong and not belong to the same object and in the same respect. [...] Of any object, one thing must be either asserted or denied." The idea of many-valued logic was initiated by Jan Łukasiewicz around 1920. "Logic changes from its very foundations if we assume that in addition to truth and falsehood there is also some third logical value or several such values" (Klement and Navara, 1999). Many-valued logic was for several decades considered as a purely theoretical topic. It was the introduction of fuzzy sets by Zadeh in 1965 (Zadeh, 1965), which opened the way to fuzzy logics.

2. Directions and goals of the research

Aggregation functions are inevitably used in fuzzy logic, as a generalization of logical connectives. In artificial intelligence, these techniques are mainly used when a system has to make a decision. It is possible that the system has not only a single criteria for each alternative, but several ones. This case corresponds to a multicriteria decision-making problem. Furthermore, if a system needs a good representation of an environment, it needs the knowledge supplied by information sources in order to be reliable. However, the information supplied by a single information source (by a single expert or sensor) is often not reliable enough. That is why the information provided from several sensors (or experts) should be combined to improve data reliability and accuracy and also to include some features that are impossible to perceive with individual sensors.

In fuzzy modeling framework, the relationship of the input and the output can be modeled by splitting the input into fuzzy regions for which we can describe the output in different ways. In several applications, the roles of the inputs are not symmetric and have different semantic contents. In this case, a proper construction of aggregation functions is needed. To fulfil this requirement, a generation method of aggregation functions from two given ones was examined in Chapter 2. The so-called threshold construction method is based on an adequate scaling on the second variable of the initial operators.

The main factor in determining the structure of the needed aggregation function is the relationship between the criteria. At one extreme there is the case in which we desire all the criteria to be satisfied. At the other extreme is the situation in which we want the satisfaction of any of the criteria. These two extreme cases lead to the use of "and" and "or" operators to combine the criteria functions. A decision can be interpreted as the intersection of fuzzy sets, usually computed by applying a t-norm based operator, when there is no compensation between low and high degrees of membership. If it is interpreted as the union of fuzzy sets, represented by a t-conorm based operator, full compensation is assumed. However, it is obvious that no managerial decision represents any of these extreme situations. As it is well-known, uninorms generalize both t-norms and t-conorms as they allow for a neutral element neutral element anywhere in the unit interval.

In Chapter 3, new construction methods of uninorms with fixed values along the borders were discussed, and sufficient and necessary conditions were presented. These results have theoretic importance in this research field.

One of the most significant problems of fuzzy set theory is the proper choice of set-theoretic operations (Schweizel and Sklar, 1983, Weber, 1983). The class of nilpotent t-norms has preferable properties which make them more usable in building up logical structures. Among these properties are the fulfillment of the law of contradiction and the excluded middle, or the coincidence of the residual and the S-implication (Dubois and Prade, 1991, Trillas and Valverde, 1985). Due to the fact that all continuous Archimedean (i.e. representable) nilpotent t-norms are isomorphic to the Łukasiewicz t-norm (Grabisch et al., 2009, Klement et al., 2000), the previously studied nilpotent systems were all isomorphic to the well-known Łukasiewicz-logic.

The new idea of using more than one generator functions gives us a chance to build up a logical system in a significantly different way. To obtain a consistent nilpotent system with the advantage of three naturally derived negations, we have thoroughly examined the necessary and sufficient conditions in Chapter 4. In the case when the natural negations do not coincide, we get a consistent nilpotent system, which is not isomorphic to Łukasiewicz logic. The fixpoints of these natural negations can be used for determining natural thresholds for different modifying words.

Our goal was to build up a logical system with the advantage of the nilpotent property and the three naturally derived negations. To get an applicable system, we have thoroughly examined the negations, conjunctions, disjunctions, implications and equivalence operators in bounded systems (Chapter 4-6).

3. New scientific results

The thesis is concerned with the development of intelligent decision models from a theoretical point of view. It covers two main topics: in Chapter 2-3, two special types of aggregation functions are studied and results on new construction methods are presented, while in Chapter 4-6, the so-called general nilpotent operator system is introduced and examined.

1. **Thesis Group I.**

   The properties of a new construction method of aggregation functions from two given ones, called threshold construction, are discussed. This class of non-symmetric functions provides a generalization of t-norms and t-conorms by partitioning the unit interval with respect to only one variable. In fuzzy modeling framework, the relationship of the input and the output can be modeled by splitting the input into fuzzy regions for which we can describe the output in different ways.

   (a) ***Thesis 1.1.*** The new type of aggregation function turned out to be monotonic and continuous, having a right-neutral and idempotent element. Three possible ways of symmetrizations are studied, two of them using min-max operators and the third using uninorms. After proving the lack of associativity in all cases, the bisymmetry and all the other associativity-like equations known from the literature are studied.

   (b) ***Thesis 1.2.*** New construction methods of uninorms with fixed values along the borders are presented. Sufficient and necessary conditions are presented.

   Relevant own publications pertaining to this thesis group: [79, 80].

1. **Thesis Group II.**

   In the second part of the thesis (Chapter 4-6), logical systems, more specifically, nilpotent logical systems are in consideration. It is shown that a consistent logical system generated by nilpotent operators is not necessarily isomorphic to Łukasiewicz-logic. This new type of nilpotent logical systems is called a bounded system, which has the advantage of three naturally derived negations. Implication and equivalence operators in bounded systems are deeply examined and a wide range of examples is also presented.

(a) ***Thesis 2.1.***

The concept of a nilpotent connective system is introduced. It is shown that a consistent logical system generated by nilpotent operators is not necessarily isomorphic to Łukasiewicz-logic, which means that nilpotent logical systems are wider than we have thought earlier. Using more than one generator functions, three naturally derived negations are examined. It is shown that the coincidence of the three negations leads back to a system which is isomorphic to Łukasiewicz-logic. Consistent nilpotent logical structures with three different negations are also provided.

(b) ***Thesis 2.2.***

Necessary and sufficient conditions for the classification property (the excluded middle and the law of contradiction), the De Morgan law and consistency have been given.

(c) ***Thesis 2.3.***

Both R- and S-implications with respect to the three naturally derived negations of the bounded system are considered. It is shown that these implications never coincide in a bounded system, as the condition of coincidence is equivalent to the coincidence of the negations, which would lead to Łukasiewicz logic. The formulae and the basic properties of four different types of implications are given, two of which fulfill all the basic properties generally required for implications. A wide range of examples is also presented. The concept of a weak ordering property is defined. Two different implications, i_c and i_d are introduced, both of which fulfill all the basic features generally required for implications.

(d) ***Thesis 2.4.***

A detailed discussion of equivalence operators in bounded systems are given. Three different types of operators are studied. After taking a closer look at the implication-based equivalences, the properties of the so-called dual equivalences are studied. Using these two types of equivalence operators, a new concept of aggregated equivalences is introduced. The paradox of the equivalence relation is solved by aggregating the implication-based equivalence and its dual operator. It is shown that the aggregated equivalence possesses nice properties like threshold transitivity, T-transitivity and associativity. For applications in image processing, the overall equivalence of two grey level images was defined, and an important semantic meaning of the aggregated equivalences is given.

Relevant own publications pertaining to this thesis group: [81, 82, 83, 84, 85].

4. Practical applicability

In fuzzy modeling framework, the relationship of the input and the output can be modeled by splitting the input into fuzzy regions for which we can describe the output in different ways. In several applications, the roles of the inputs are not symmetric and have different semantic contents. In this case, the introduced threshold construction of aggregation functions has great importance.

By examining nilpotent logival systems, our goal was to build up a consistent system with the advantage of the nilpotent property and the three naturally derived negations. The fixpoints of these natural negations can be used for determining natural thresholds for different modifying words. To get a widely applicable system, we have thoroughly examined the negations, conjunctions, disjunctions, implications and equivalence operators in bounded systems. For applications in image processing, the overall equivalence of two grey level images was defined, and an important semantic meaning of the aggregated equivalences was given.

The main disadvantage of the Łukasiewicz operator family is the lack of differentiability, which would be necessary for numerous practical applications. Although most fuzzy applications (e.g. embedded fuzzy control) use piecewise linear membership functions due to their easy handling, there are significant areas, where the parameters are learned by a gradient based optimization method. In this case, the lack of continuous derivatives makes the application impossible. For example, the membership functions have to be differentiable for every input in order to fine tune a fuzzy control system by a simple gradient based technique.

This problem could be easily solved by using the so-called squashing function (see Dombi and Gera, 2008), which provides a solution to the above mentioned problem by a continuously differentiable approximation of the cut function. This approximation could provide the next step along the path to a practical and widely applicable system, with the advantage of three naturally derived negation operators.

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