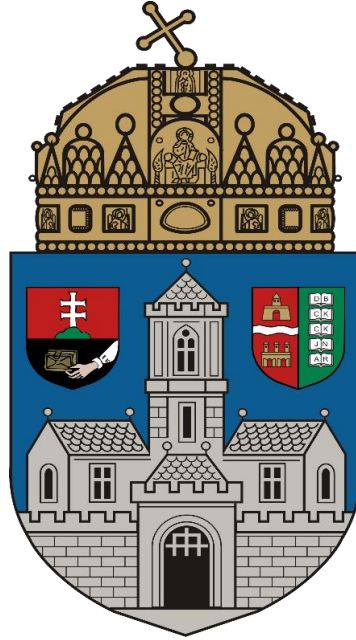


Óbuda University
PhD Thesis Booklet



**Advanced Application of the Catenary and the Parabola for
Mathematical Modelling of the Conductor and Sag Curves
in the Span of an Overhead Line**

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Contents

	Page
Introduction	3
Motivation	3
Structure of the Dissertation and Research Objectives	4
Methodology of the Dissertation	6
Application of the Catenary Model	8
Thesis 1	8
Inclined Span Modelling by a Given Level Span	11
Thesis 2	11
Application of the Parabola Model	12
Thesis 3	12
Universal Formulas for the Conductor Length	14
Thesis 4	14
Practical Application of the New Results	15
Own Publications	16
Journal Papers Related to the Theses	16
Conference Papers Related to the Theses	18
Conference Papers not Related to the Theses	20

Introduction

Electrical network can be divided into two basic groups, underground cables and overhead lines (OHL). It is well known that the construction of OHL is less expensive but its design is more complex. One of the reasons of the latter disadvantageous fact is the conductor sag, which directly affects clearance calculations. The OHL have to be designed and operated so that they would not cause injuries to people, therefore maintaining adequate distance between energized conductors and ground or other objects is a particularly important task of OHL design. When designing electrical network a special attention should be paid to the safety of its environment. My dissertation has been written in this spirit.

Focusing on the conductor sag in a span, this dissertation introduces novel methods, algorithms and equations, which are creatable or obtainable by the use of the given major result of the sag–tension calculation (catenary parameter or parabola’s maximum sag), besides the span length and the heights of the support points. Both the catenary and the parabola based calculations have been discussed, as well as the special link between them, providing a wide mathematical background which can help to solve not only standard and frequent tasks in OHL practice, but also some rare unconventional ones. This work is a complex mathematical module, which practically connects the results of the sag–tension calculation with clearance calculation, and in this way it contributes to safe electrical network planning.

Motivation

My interest toward electrical gadgets and electrical energy comes from my childhood; it was evident for me that I was going to be an electrical engineer. Thus, I attended and finished Electrical Engineering High School and graduated from the Faculty of Electrical Engineering. Later I got a job in a company whose main activities were designing and constructing electrical distribution network. Working as an electrical designer engineer I faced some generally used approximate methods and calculations for OHL design. In fact, I have concluded that the sag–tension calculation is appropriate and well

described in literatures, but new mathematical equations related to the conductor curve and the sag are needed for an easy and correct determination of the conductor height and the sag at any point of the span, without any limitations caused by the span inclination or the span type. This is of a particularly high importance for accurate clearance calculation. Hence, I started to create new methods and mathematically exact relations in order to complement the current OHL design. This way, come-by-chance, I found a field of my future research. The motivation was double, providing novel results and taking entire responsibility for planned network. I have been working as an electrical design engineer and later as a senior engineer for electric network development for more than 20 years. Having had an opportunity to plan and design kilometres of overhead lines and underground cables and gaining a lot of experience during my work, I could also provide many new methods and calculations and then check them in practice. As a result of that work I have written several scientific publications for respectable professional conferences and scientific journals in both Hungary and abroad.

Structure of the Dissertation and Research Objectives

As I spent many years doing different activities connected to OHL design both as an electrical design engineer and also as a plan supervisor, I had the opportunity to recognize the special problems, which electrical design engineers face in practice when planning OHL, but literatures does not give adequate solutions. In accordance to that, my aim was to complement the OHL design by special new equations for an easy application in practice. On the other hand, I wanted to provide a mathematical background, which explains not only the derivation of the new equations but also the behaviour of the parabola and the catenary when the span inclination changes, as well as a mathematical connection between the two latter curves. These are important conditions for a conscious design. The main objectives are the following:

- The aim of Chapter 1 was to present the drone and its wide usability for overhead lines inspection. The necessity of the mathematical

algorithms (derived in Chapters 2–5) is highlighted for planning the appropriate flight path of an autonomous drone applied for inspection of overhead lines.

- Considering the conductor curve as a catenary, the goal of Chapter 2 was to derive universal equations for determining the conductor height and the sag at any point of the span, usable in all span types, applying the coordinate system in a new way, which is uniformly used through the whole dissertation.
- The aim of Chapter 3 was to create a method for modelling an inclined span by known data of a level span, when the span length and the catenary parameter are common data in both spans. Evaluating the use of $1/\cos\psi$ multiplier in the case of the catenary was also targeted.
- In Chapter 4 the goal was to derive universal parabolic equations for the conductor and the sag curves, and to create a mathematical parabolic approximation of the catenary in an inclined span and then to evaluate the application of $1/\cos\psi$ in the case of the parabola.
- Chapter 5 deals with the conductor length calculation separately for the cases of the parabola and the catenary, and also gives the comparison of the lengths of the catenary and its approximation by a parabola, both the basic and the modified ones, i.e. first without and then also with the use of $1/\cos\psi$ multiplier.
- The aim of Chapter 6 was to introduce the extension of the new methods shown in Chapters 2–5 in the entire section of OHL consisting of several support spans between the two dead-end structures.

It is important to emphasize that the mathematical background has been provided in such a way that the frequently applied conveniences for simplification of calculations, as for instance assuming that the maximum sag of the catenary in an inclined span is located at a mid-span or using $1/\cos\psi$

multiplier in the case of the catenary, are absolutely avoided. All new equations are mathematically exact ones without simplifications and are accompanied by appropriate explanations. The main steps of the derivations have been given within the body of the chapters, but deductions of significant lengths have been presented separately in Appendices, in order to read the dissertation easily.

Chapters 2–5 have a very similar structure. Each one starts with an introduction of the actual subject and gives a literature overview highlighting those with a particular importance or uniqueness in the field of the actual research. The biggest part of the chapter is about the achievement of the new results and their explanation in details. A practical application has been shown through numerical examples, which are commonly used also for analysis of results and drawing important conclusions. Ending a chapter, the new results are summarised.

Methodology of the Dissertation

In order to describe the conductor curve when planning overhead lines, the parabola or the catenary model is used. Thus, the calculations can be parabola or catenary based. The last one is known as exact and complicated, while the first one as approximate and simple. In practice it is a well-known fact that when the spans are large (for instance over 400 metres) the conductor curve cannot be considered as a parabola, since the difference in comparison to the catenary is then not negligible. According to this work, when the conductor curve is considered as a catenary, then the main datum, which has to be taken from the sag–tension calculation, is a parameter of the catenary, while in the case of the parabola it is the maximum sag. Since both cases have been discussed separately, the main input data (all given in metres) are grouped as follows, while their usage is detailed in Chapters 2 and 4.

Catenary

S – span length

h_1 – height of the left–hand side support point

Parabola

S – span length

h_1 – height of the left–hand side support point

h_2 – height of the right–hand side support point
 c – catenary parameter

h_2 – height of the right–hand side support point
 D_{\max} – maximum sag of the parabola

The first three data listed above are considered as known or given ones, while the fourth datum is taken from the sag–tension calculation. The latter calculation is not the subject of this work, as it is widely available and well explained in literature. The focus is placed on deriving new equations for the conductor curve and the sag, and also defining the length formulas.

Generally, each calculation is referred to one temperature of the conductor, and it is the one which the catenary parameter or the parabola’s maximum sag datum is related to. The change in temperature causes the change of the two latter data, and hence the conductor curve is different at each temperature, as well as the conductor sag and the length. In accordance with that, the minimum ground clearance have to be checked for the most unfavourable conditions (worst–case scenario), i.e. when the sag is maximum (see in Fig. 1). However, it is worth mentioning that ice load can also be the cause of the maximum sag.

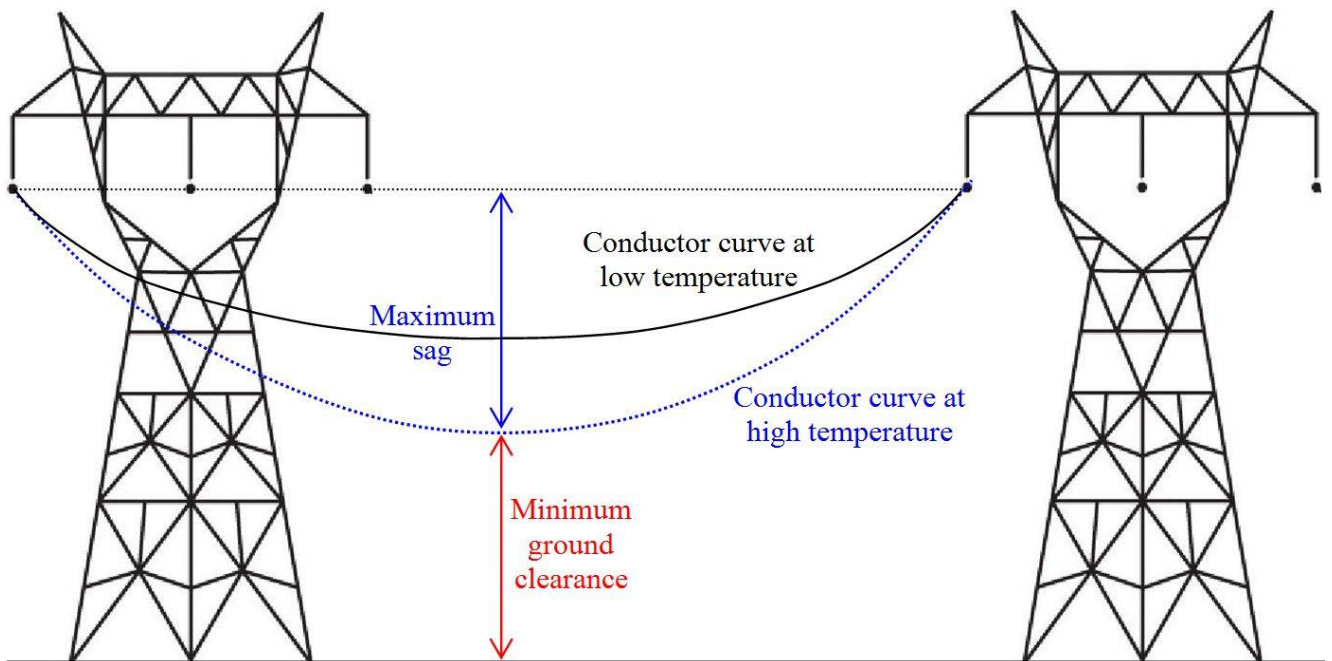


Fig. 1: Conductor curve, maximum sag and minimum ground clearance

Determining the equation for the conductor curve is of high importance, because the conductor height then can be calculated at any point of the span. It is necessary for instance to calculate the conductor clearance when some

objects are placed under the conductors in a span. Fig. 1 shows a simple example with the supports on the same elevation. It is a level span. When the supports are on the different elevations (inclined span) each calculation becomes more difficult. In that case the lowest point of the conductor is not located at a mid-span, but it is removed. This work has targeted inclined spans, but level ones have been discussed as well.

All new equations and relations in this work have been derived analytically and most of them have directly been checked in practice and by practical numerical examples as well. Knowing that the parabola, the catenary and the square of hyperbolic sine are all even functions, they are suitably applied for creating the *mirror image* examples, which made the proposed methods and also the correctness of the obtained results very clear. There are practical examples in each chapter used also for drawing important conclusions, which cannot be drawn analytically. Considering the fact that the parabola is an algebraic function, while the catenary is a transcendental one, therefore solutions of both algebraic and transcendental equations are included. The parabola based calculation is improved by algebraic transformations and matrix calculus, while the catenary based calculation is widened by the application of hyperbolic and their inverse functions, as well as related identities. Basic mathematical techniques for finding the first derivative and the maximum of the curve have been applied in both cases. New equations have been derived for the use in inclined spans, whereas the adequate equations related to level spans have been defined as the simplifications of the first ones. The conductor length calculation is uniformly improved by the use of the integral calculus.

Application of the Catenary Model

Thesis 1.

Relating to the drawing of the conductor curve considered as a catenary, I have derived universal equations for the conductor and the sag curves which are applicable for determining the conductor height and the sag at any point of the span, in all possible span types with any span inclination. New equations

also cover the special cases of inclined spans where the catenary's vertex point and the conductor's low point differ in their location.

Universal equation for the conductor curve:

$$y(x) = 2c \cdot \left\{ \sinh^2 \left[\frac{S}{4c} - \frac{x}{2c} - \frac{1}{2} \operatorname{arcsinh} \frac{h_2 - h_1}{2c \cdot \sinh(S/2c)} \right] - \sinh^2 \left[\frac{S}{4c} - \frac{1}{2} \operatorname{arcsinh} \frac{h_2 - h_1}{2c \cdot \sinh(S/2c)} \right] \right\} + h_1 \quad x \in [0, S]$$

Universal equation for the sag curve:

$$D(x) = \frac{h_2 - h_1}{S} x - 2c \cdot \left\{ \sinh^2 \left[\frac{S}{4c} - \frac{x}{2c} - \frac{1}{2} \operatorname{arcsinh} \frac{h_2 - h_1}{2c \cdot \sinh(S/2c)} \right] - \sinh^2 \left[\frac{S}{4c} - \frac{1}{2} \operatorname{arcsinh} \frac{h_2 - h_1}{2c \cdot \sinh(S/2c)} \right] \right\} \quad x \in [0, S]$$

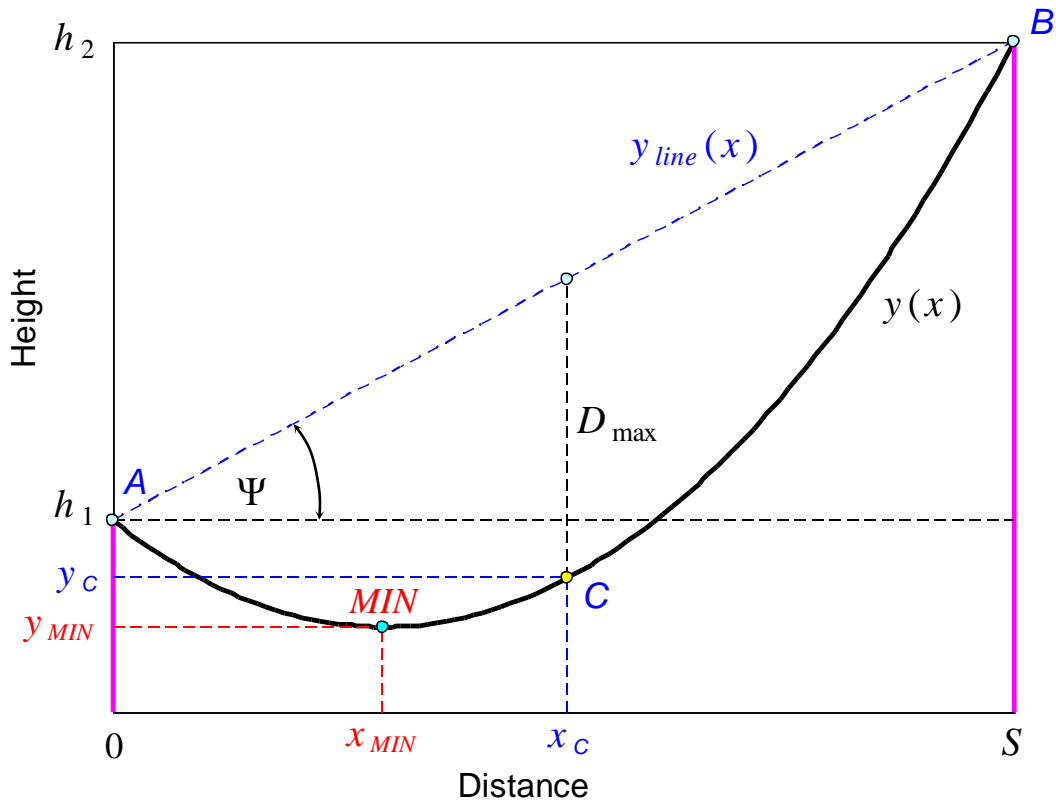


Fig. 2: Catenary conductor curve in an inclined span

I have shown that the new sag equation can be used for determining the location of the maximum sag in a span and also for deriving the special

formulas for the characteristic sags: the maximum sag, the mid–span sag and the low point sag.

Maximum sag formula:

$$D_{\max} = 2c \left\{ \frac{h_2 - h_1}{2S} \left[\frac{S}{2c} - \operatorname{arcsinh} \frac{h_2 - h_1}{2c \cdot \sinh(S/2c)} + \operatorname{arcsinh} \frac{h_2 - h_1}{S} \right] - \sinh^2 \left(\frac{1}{2} \operatorname{arcsinh} \frac{h_2 - h_1}{S} \right) + \sinh^2 \left[\frac{S}{4c} - \frac{1}{2} \operatorname{arcsinh} \frac{h_2 - h_1}{2c \cdot \sinh(S/2c)} \right] \right\}$$

Mid–span sag formula:

$$D(S/2) = \frac{h_2 - h_1}{2} - 2c \cdot \left\{ \sinh^2 \left[\frac{1}{2} \operatorname{arcsinh} \frac{h_2 - h_1}{2c \cdot \sinh(S/2c)} \right] - \sinh^2 \left[\frac{S}{4c} - \frac{1}{2} \operatorname{arcsinh} \frac{h_2 - h_1}{2c \cdot \sinh(S/2c)} \right] \right\}$$

Low point sag formula:

$$D(x_{\min}) = 2c \cdot \left\{ \frac{h_2 - h_1}{2S} \left[\frac{S}{2c} - \operatorname{arcsinh} \frac{h_2 - h_1}{2c \cdot \sinh(S/2c)} \right] + \sinh^2 \left[\frac{S}{4c} - \frac{1}{2} \operatorname{arcsinh} \frac{h_2 - h_1}{2c \cdot \sinh(S/2c)} \right] \right\} \quad \forall \quad 0 \leq x_{\min} \leq S$$

I have demonstrated that the direction of the movement of the maximum sag from the mid–span, which occurs when the level span changes into an inclined one, can be determined analytically, not only numerically.

I have shown that if the conductor curve is considered as a catenary, then the sag function $D(x+S/2)$ is an even function in the case of a level span, while in inclined spans it is neither an even nor an odd function. The sag curve in a level span has the exact shape of an inverted catenary, while in an inclined span it slightly differs. The difference increases with the span inclination.

Publications connected to this thesis: [S1], [S4], [S7], [S8], [S13], [S15], [S17], [S24].

Inclined Span Modelling by a Given Level Span

Thesis 2.

I have developed a mathematical method, called *inclined span modelling by a given level span*, which using the given data (S, c, h_1) for a level span and a freely chosen datum of the difference in the support points elevation $(h_2 - h_1)$, creates equations for both the conductor and the sag curves in a modelled inclined span when the span length and the catenary parameter are common data in both spans.

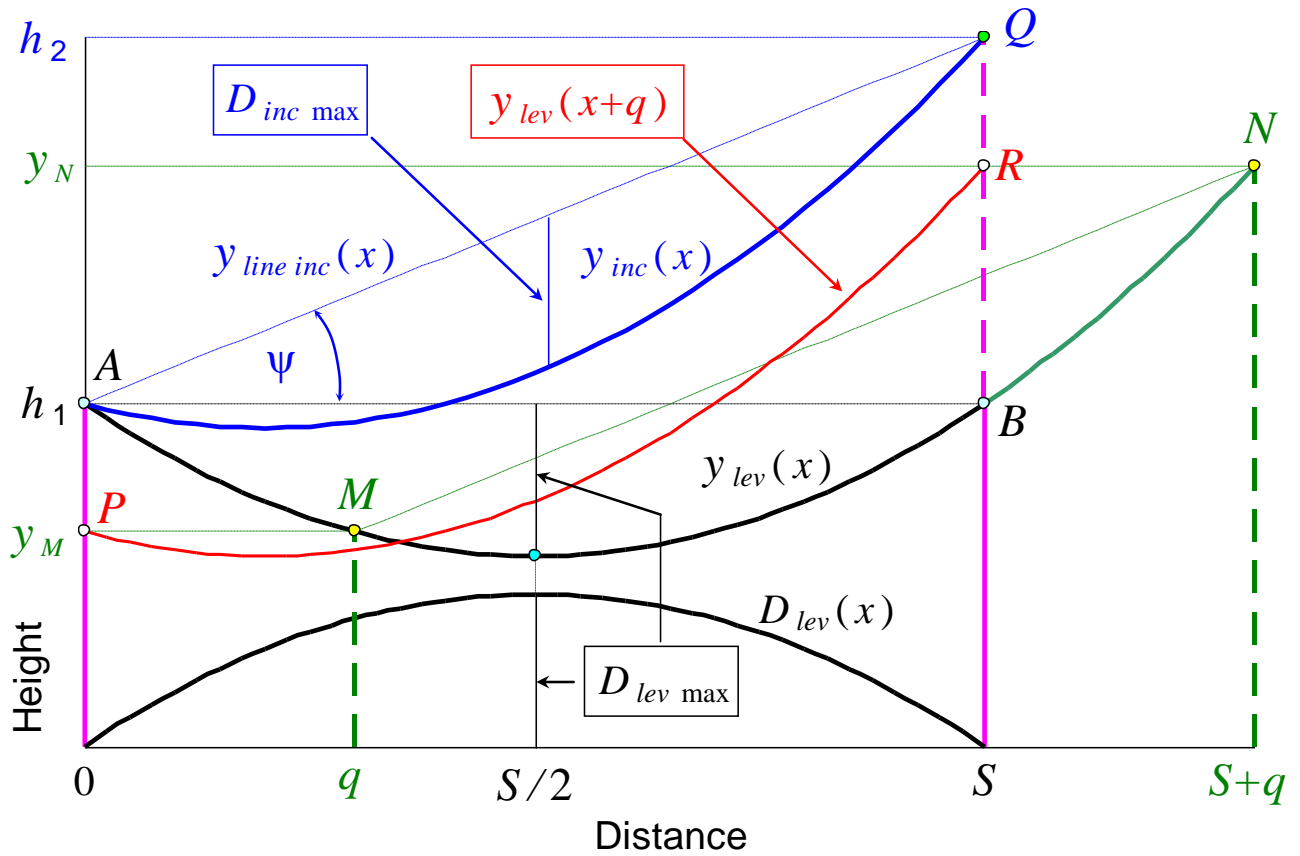


Fig. 3: Curves for explaining the *inclined span modelling by a given level span*

$$y_{inc}(x) = 2c \cdot \sinh \frac{x}{2c} \cdot \sinh \left(\frac{x-S}{2c} + \operatorname{arcsinh} \frac{h_2 - h_1}{2c \cdot \sinh(S/2c)} \right) + h_1 \quad x \in [0, S]$$

$$D_{inc}(x) = \frac{h_2 - h_1}{S} x - 2c \cdot \sinh \frac{x}{2c} \cdot \sinh \left(\frac{x-S}{2c} + \operatorname{arcsinh} \frac{h_2 - h_1}{2c \cdot \sinh(S/2c)} \right) \quad x \in [0, S]$$

I have revealed that the quotient of the sag functions in inclined and level spans on the interval (0,S) is not a constant in the case of the catenary as it is in the case of the parabola.

$$\frac{D_{inc}^{(cat)}(x)}{D_{lev}^{(cat)}(x)} = \left(\frac{D_{inc}^{(cat)}}{D_{lev}^{(cat)}} \right)(x) \neq \text{const.} \quad 0 < x < S$$

Instead of the existing approximate relation I have derived a mathematically exact one between the catenary sags in inclined and level spans, usable at any point of the span.

$$D_{inc}(x) = D_{lev}(x) + \frac{h_2 - h_1}{S} x - 4c \cdot \sinh \frac{x}{2c} \cdot \sinh \left(\frac{1}{2} \operatorname{arcsinh} \frac{h_2 - h_1}{2c \cdot \sinh \frac{S}{2c}} \right) \cdot \cosh \left(\frac{S - x}{2c} - \frac{1}{2} \operatorname{arcsinh} \frac{h_2 - h_1}{2c \cdot \sinh \frac{S}{2c}} \right)$$

The function, which describes the error along the span, resulted by the application of the approximate relation, changes sign near the middle of the span.

Instead of the existing approximate relation I have derived a mathematically exact one between the maximum sags of the catenary in inclined and level spans. The difference between the two mentioned sags increases with the span inclination.

$$D_{inc\max} = D_{lev\max} + \frac{h_2 - h_1}{S} \cdot \left(x_{MIN} + c \cdot \operatorname{arcsinh} \frac{h_2 - h_1}{S} \right) - 2c \cdot \sinh^2 \frac{S}{4c} + 2c \cdot \sinh \left(\frac{1}{2c} \left(x_{MIN} + c \cdot \operatorname{arcsinh} \frac{h_2 - h_1}{S} \right) \right) \cdot \sinh \left(\frac{1}{2c} \left(x_{MIN} - c \cdot \operatorname{arcsinh} \frac{h_2 - h_1}{S} \right) \right)$$

where x_{MIN} is the x -coordinate of the catenary's vertex point given as

$$x_{MIN} = \frac{S}{2} - c \cdot \operatorname{arcsinh} \frac{h_2 - h_1}{2c \cdot \sinh(S/2c)}$$

Publications connected to this thesis: [S2], [S8], [S9], [S10], [S14], [S17], [S25].

Application of the Parabola Model

Thesis 3.

I have derived a universal parabolic equation for the conductor curve by the given maximum sag and the coordinates of the support points, which is usable in level and inclined (classical and special) spans as well and from which the coordinates of the vertex point are directly readable.

$$y(x) = \frac{4D_{\max}}{S^2} \left[x - \frac{S}{2} \left(1 - \frac{h_2 - h_1}{4D_{\max}} \right) \right]^2 + h_1 - D_{\max} \left(1 - \frac{h_2 - h_1}{4D_{\max}} \right)^2 \quad x \in [0, S]$$

Subthesis 3.1.

I have derived special parabolic equations for the conductor curve applicable strictly in inclined spans, by the given $(x; y)$ coordinates of the two support points and only one coordinate of the vertex point, x_{MIN} or y_{MIN} .

$$y(x) = \frac{h_2 - h_1}{S(S - 2x_{MIN})} (x - x_{MIN})^2 + h_1 - \frac{(h_2 - h_1)x_{MIN}^2}{S(S - 2x_{MIN})} \quad \forall \quad h_1 \neq h_2 \quad \wedge \quad x \in [0, S]$$

$$y(x) = \left[\frac{h_2 - h_1}{S(\sqrt{h_2 - y_{MIN}} - \sqrt{h_1 - y_{MIN}})} \right]^2 \cdot \left[x - \frac{S(h_1 - y_{MIN})}{h_2 - h_1} \left(\sqrt{\frac{h_2 - y_{MIN}}{h_1 - y_{MIN}}} - 1 \right) \right]^2 + y_{MIN}$$

$$\forall \quad h_1 \neq h_2 \quad \wedge \quad x \in [0, S]$$

Subthesis 3.2.

I have created an analytical method for a parabolic approximation of the catenary in inclined spans. This method can also be applied in level ones.

$$y_{inc\psi}(x) = \frac{1}{2c \cdot \cos \psi} \left\{ x - \frac{S}{2} \left[1 - \frac{2c(h_2 - h_1)}{S^2} \cos \psi \right] \right\}^2 +$$

$$+ h_1 - \frac{1}{2c \cdot \cos \psi} \left\{ \frac{S}{2} \left[1 - \frac{2c(h_2 - h_1)}{S^2} \cos \psi \right] \right\}^2 \quad x \in [0, S]$$

I have provided a wide mathematical background which is related to $1/\cos\psi$ multiplier's effect. As the sag in an inclined span increases, $1/\cos\psi$ multiplier reduces the parabola's parameter and also its deviation from the catenary,

resulting that the modified parabola resembles the catenary better than the basic (original) parabola.

$$\left| y_{\text{par } \psi}(x) - y_{\text{cat}}(x) \right| < y_{\text{par}}(x) - y_{\text{cat}}(x) \quad \forall \quad 0 < x < S \quad \wedge \quad h_1 \neq h_2$$

Subthesis 3.3.

I have revealed that differently from the case of the catenary, the quotient of the sag functions in inclined and level spans on the interval $(0,S)$ is a constant in the case of the parabola (either basic or modified by $1/\cos\psi$), due to the two following relations:

$$\frac{D_{inc}^{(\text{par})}(x)}{D_{lev}^{(\text{par})}(x)} = \left(\frac{D_{inc}^{(\text{par})}}{D_{lev}^{(\text{par})}} \right)(x) = 1 \quad 0 < x < S$$

$$\frac{D_{inc\psi}^{(\text{par}\psi)}(x)}{D_{lev}^{(\text{par})}(x)} = \left(\frac{D_{inc\psi}^{(\text{par}\psi)}}{D_{lev}^{(\text{par})}} \right)(x) = \frac{1}{\cos \psi} \quad 0 < x < S$$

Publications connected to this thesis: [S4], [S5], [S6], [S15], [S16], [S17], [S18], [S20], [S22], [S23], [S26], [S27], [S28], [S29].

Universal Formulas for the Conductor Length

Thesis 4.

I have derived one universal formula for computing the length of the parabola and one for computing the length of the catenary, which are both usable in inclined and level spans as well, in full span and also in its arbitrarily chosen part.

Universal formula for the length of the parabola:

$$\begin{aligned}
 L_{x_1 x_2} = & \frac{S^2}{16D_{\max}} \operatorname{arcsinh} \left(\frac{8D_{\max}}{S^2} \left(x_2 - \frac{S}{2} \left(1 - \frac{h_2 - h_1}{4D_{\max}} \right) \right) \right) - \\
 & - \frac{S^2}{16D_{\max}} \operatorname{arcsinh} \left(\frac{8D_{\max}}{S^2} \left(x_1 - \frac{S}{2} \left(1 - \frac{h_2 - h_1}{4D_{\max}} \right) \right) \right) + \\
 & + \frac{1}{2} \left(x_2 - \frac{S}{2} \left(1 - \frac{h_2 - h_1}{4D_{\max}} \right) \right) \cdot \sqrt{1 + \left(\frac{8D_{\max}}{S^2} \left(x_2 - \frac{S}{2} \left(1 - \frac{h_2 - h_1}{4D_{\max}} \right) \right) \right)^2} - \\
 & - \frac{1}{2} \left(x_1 - \frac{S}{2} \left(1 - \frac{h_2 - h_1}{4D_{\max}} \right) \right) \cdot \sqrt{1 + \left(\frac{8D_{\max}}{S^2} \left(x_1 - \frac{S}{2} \left(1 - \frac{h_2 - h_1}{4D_{\max}} \right) \right) \right)^2}
 \end{aligned}$$

Universal formula for the length of the catenary:

$$L_{x_1 x_2} = 2c \cdot \sinh \frac{x_2 - x_1}{2c} \cdot \cosh \left(\frac{x_1 + x_2 - S}{2c} + \operatorname{arcsinh} \frac{h_2 - h_1}{2c \cdot \sinh(S/2c)} \right)$$

Subthesis 4.1.

Related to OHL practice, I have shown that when calculating the conductor length, the application of multiplier $1/\cos\psi$ for modifying the basic parabola in inclined spans ensures results closer to the catenary length in comparison to the case when the multiplier is not applied.

$$L_{\text{cat}}^{(inc)} > L_{\text{par}\psi}^{(inc)} > L_{\text{par}}^{(inc)}$$

Subthesis 4.2.

Related to OHL practice, I have revealed that when the span inclination (or $|h_2 - h_1|$) increases, then the difference between the lengths of the catenary and its approximation by the modified parabola decreases, whereas the difference between the lengths of the catenary and its approximation by the basic parabola increases. It is expressed mathematically in the following two relations with the use of $|h_2 - h_1|$:

$$\begin{aligned}
 L_{\text{cat } 2}^{(inc)} - L_{\text{par}\psi 2}^{(inc)} & < L_{\text{cat } 1}^{(inc)} - L_{\text{par}\psi 1}^{(inc)} \quad \forall \quad |h_2^{(2)} - h_1^{(2)}| > |h_2^{(1)} - h_1^{(1)}| \\
 L_{\text{cat } 2}^{(inc)} - L_{\text{par } 2}^{(inc)} & > L_{\text{cat } 1}^{(inc)} - L_{\text{par } 1}^{(inc)} \quad \forall \quad |h_2^{(2)} - h_1^{(2)}| > |h_2^{(1)} - h_1^{(1)}|
 \end{aligned}$$

Publications connected to this thesis: [S3], [S4], [S11], [S12], [S19], [S21], [S29].

Practical Application of the New Results

The practical usage of the new results is well presented and described through suitable numerical examples given in Chapters 2–5. The main application is the determination of the conductor height and the sag at any point of the span, by the universal equations for the conductor and the sag curves, which are applicable in any span type with any span inclination. New equations have been derived in the case of the parabola and the catenary as well. Drawing the conductor curve is another application of high importance. Besides computing all characteristic sags of the catenary, the inclined span modelling, the conductor length calculation and the parabolic approximation of the catenary in an inclined span are also very useful results presented in this work. The main new results were introduced in practice. My former company accepted my methods and implemented them in OHL design process after I had trained the other designers in the company. Representing the previous company, my project named *Designer Programme* successfully entered the *19th Hungarian Innovation Award Competition* and as a recognised innovation got into the *Innovation Award 2010* book under number 12.

The future application of the new mathematical equations and algorithms presented in the dissertation is their implementation in planning the trajectory of an autonomous drone used for inspection of overhead lines and for mounting, maintaining or replacing the smart sensors.

Own Publications

Used languages: *English, Hungarian, Croatian/Bosnian/Serbian*

Journal Papers Related to the Theses:

[S1] Alen Hatibovic: *Derivation of Equations for Conductor and Sag Curves of an Overhead Line Based on a Given Catenary Constant*, PERIODICA POLYTECHNICA ELECTRICAL ENGINEERING, Vol. 58, No. 1, pp. 23–27, 2014, ISSN 2064–5260, DOI: 10.3311/PPee.6993

- [S2] Alen Hatibovic: *Inclined Span Modelling by a Given Levelled Span for OHL Design*, PERIODICA POLYTECHNICA ELECTRICAL ENGINEERING, Vol. 58, No. 2, pp. 69–79, 2014, ISSN 2064–5260, DOI: 10.3311/PPee.7373
- [S3] Alen Hatibovic: *Integral Calculus Usage for Conductor Length Determination on the Basis of Known Maximal Sag of a Parabola*, PERIODICA POLYTECHNICA ELECTRICAL ENGINEERING, Vol. 56, No. 2, pp. 35–41, 2012, ISSN 2064–5260, DOI: 10.3311/PPee.7076
- [S4] Alen Hatibovic: *Parabola and Catenary Equations for Conductor Height Calculation*, ELECTROTEHNICA ELECTRONICA AUTOMATICA, Vol. 60, No. 3, ISSN 1582–5175, pp. 22–28, 2012
- [S5] Alen Hatibovic: *Analysis of the Quadratic Equation for the Conductor Sag Based on a Given Maximal Sag*, BOSANSKOHERCEGOVAČKA ELEKTROTEHNIKA 7, ISSN 1515–5483, pp. 22–28, 2013
- [S6] Alen Hatibovic: *Vezetékgörbe egyenletének meghatározása a parabola ismert legnagyobb belógása alapján*, Elektrotechnika 2012/10, Vol. 105, ISSN 0367–0708, pp. 20–23
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