Óbuda University

PhD Thesis Booklet

Tools for Efficient Soft Computing Modelling and Feasible Optimal Control of Complex Dynamic Systems,

with Application to Multi-Rotor Unmanned Aerial Vehicle Navigation and Obstacle Avoidance

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CONTENTS

INTRODUCTION

Formulation of the Studied Scientific Problem

The focus for this dissertation is on studying mechanical systems of complex dynamics. Unmanned aerial vehicles and robotic manipulators are typical examples of complex dynamics systems. The difficulty in wide spread studding robotic manipulators lies in their relative low availability and high cost. A wide area of robotics research is dedicated to aerial platforms, which have very similar dynamics and are more simple to build and also commercially available in wide ranges. Versatile flying structures and configurations have been developed to allow 3D movements [35], [60]. For example, there are blimps, fixed-wing planes, single rotor helicopters, bird-like prototypes, quadrotors, hexa-rotors, octa-rotors, etc. Each of these has advantages and drawbacks. The vertical take-off and landing (VTOL) requirements exclude some of the aforementioned configurations.

The quadrotor architecture has low dimensions, good manoeuvrability, simple mechanics and good payload capability. The main drawback is the relatively high energy consumption and difficult precision flight control; however, the trade-off results are very positive. This structure can be attractive in several applications, in particular for surveillance, for imaging dangerous environments, and for outdoor navigation and mapping. The study of kinematics and dynamics helps to understand the flight mechanics of the quadrotor and its behaviour [33], [12]. Together with system modelling, the definition of the control algorithm structure is very important. Soft computing methods can be efficiently applied together with, and even instead of conventional controllers [63].

Multi-rotors like quad- and hexa-rotors are popular representatives of VTOL unmanned aerial vehicles (UAVs) as they are relatively simple to build, while being of versatile applicability, also capable of vertical take-off and landing. Also the multi-rotor architecture has simple mechanics, high relative payload capability and good manoeuvrability. The study of multi-rotor kinematics and flight dynamics is based on the physics of aerial platforms - flying bodies, a good description of such can be found in [35]. The kinematics and general force and torque dynamics, flight mechanics of any symmetric multi-rotor (quad-, hexa- or any other number of rotors) is equivalent.

This work presents an efficient toolset improvement proposal for multi-rotor aerial vehicles control system design. Efficient autonomous navigation and obstacle avoidance requires a fast, direct method for calculating time and energy efficient feasible trajectories. Efficient control systems in real-life outdoor environment require robust adaptive system models. Designing robust fuzzy systems require efficient global and precise local optimisation techniques.

The first part of this theses collection proposes improvements of multi-objective stochastic search by a new vector comparison ordered inequality operator and ranking method. The efficiency analysis of the new method is presented on well-studied genetic algorithms of proven convergence capability and carefully designed, mathematically sound, difficult multi-criteria optimisation problems of various Pareto-front form and search space density.

The second part proposes a novel representation of fuzzy-partitions based inference systems for universal function approximations. The proposed fuzzy-partition parameter representation of non-linear parameters of these systems makes it possible to be subjected to efficient unconstrained optimisations by global search algorithms and fine-tuning with gradient descent based methods. Linear parameters of these fuzzy-systems are best calculated based on singular value decomposition to achieve mean square error minimisation. The new multi-objective stochastic search methods introduced in the first part are used to find non-dominated fuzzy system solutions where both the fuzzy structure complexity, the number of membership functions and rules, and the function approximation error, both the absolute maximum error and the mean square sum of the error is first globally minimised then locally fine-tuned by a gradient descent method.

The third part proposes a new method for robust fuzzy-system based modelling of complex dynamic systems as robot manipulators and mobile robots, like free flying multi-rotors. For the six degree of freedom multi-rotors a special extension is proposed for periodic continuous extension of fuzzy systems. This new method follows the grey box identification approach, making use of well-known system properties of robotic manipulators. My proposal results in a system approximation, which has all the benefits of robust fuzzy systems, and also manifests all the analytical properties of dynamic models that are used for analysing system and system control properties. Application of these fuzzy system based grey box models in classical hard computing control techniques is straightforward, as it is possible to explicitly analytically extract all system states and their derivatives.

The fourth part introduces a direct, iteration free single-pass algorithm using simple closed formulas, to design time and energy efficient trajectory parametrisations with pre-defined time derivative constraints. The method enables designing trajectories tuned to system (including control actuator) capabilities and ensuring oscillations free control possibilities. The method is presented for both multi-rotor trajectory designs, where higher derivative smoothness is a must for efficient control, and it is also presented for 3D overhead crane trajectory designs to analyse its oscillations free property.

The fifth part presents a new method for fuzzy-system training data set reduction.

Research Objectives

As concluded in the introductory chapter for efficient multi-rotor autonomous navigation and obstacle avoidance improvement it is necessary to master the following design and engineering tools:

1. *Efficient multi-objective search*

My first goal is to define a new operator for comparing two vectors, which can be used as basis for an efficient multi-objective ranking method for Genetic Algorithm (GA), which performs better than the existing Pareto dominance based algorithms.

2. *Efficient genetic fuzzy system universal function approximation*

My second goal is to define a new method for an efficient unconstrained optimisation of Takagi-Sugeno-Kang (TSK) Fuzzy Logic Systems (FLS) subject to both a GA based global search and further ANFIS like gradient descent based local fine tuning of fuzzy partition antecedent Membership Function (MF) parameters. The MF rule base has to remain intact; complete fuzzy partitions have to remain with keeping the pre-defined linguistic variable order. The resulting FLS has to be capable of acting as a universal function approximation.

3. *Efficient robust modelling method for autonomous control of complex systems of nonlinear dynamics*

My third goal is to define a new efficient robust system dynamics modelling method, which results in a system model that can be readily used for efficient autonomous, system state model based control of complex nonlinear dynamics systems such as robot manipulators (RM) and multi-rotor unmanned aerial vehicles (UAV) navigation dynamics.

4. *Efficient trajectory design method for autonomous control of complex nonlinear dynamic systems*

My fourth goal is to define a new method for an efficient real-time direct path parametrisation design algorithm for generating physically feasible, time-and energy optimal, bounded, continuous trajectories that induce no system oscillations. The notion of time and energy optimality is not to be used in some mathematics theory manner but in real life physically feasible engineering manner. Finding optimal trajectories is focused on finding the appropriate parametrisation for the path vector function, given the pre-defined feasibility limits on the displacement time derivatives.

5. *Efficient genetic fuzzy system training data set reduction method*

My fifth goal is to define a new method for an efficient genetic fuzzy system (GFS) training data set reduction, which will significantly reduce the data set size, while maintaining the quality of the identification process, and thus significantly increase the identification process performance, independent of the system to be identified.

Research Hypothesis

Hypothesis I: there exists a vector comparison method which is capable of guiding a multi-objective stochastic search more efficiently than the Pareto dominance relation.

Hypothesis II: there exists a more suitable parametrisation method for antecedent fuzzy partition MF components of a TSK FLS, which is still simple to directly compute and optimise without any restrictions, both by stochastic search and/or with gradient descent methods; and for every case the formed parameters will inherently satisfy all of the required constraints for a stable fuzzy partition antecedent structure, keeping the associated linguistic values.

Hypothesis III.a: the singular value decomposition (SVD) algorithm is efficient enough **to extract each basic component of a dynamic system described by Euler-Lagrange** equation, when there are a sufficient number of good quality training samples available. Further on the nonlinear inertia component functions can be robustly identified with TSK FLSs, while the nonlinear functions describing the centrifugal and Coriolis effects can be exactly derived from the identified TSK FLSs for the inertia components. The nonlinear parameters of TSK FLSs modelling a Robotic Manipulator (RM) dynamic system can be efficiently found by a multi-objective hybrid GA together with gradient descent method fine-tuning, while all the linear parameters of the used TSK FLSs, including constants of the model can be directly calculated with SVD based robust least squares (LS) method. The RM trajectory used for collecting training samples have to be sufficiently exciting to reveal all the characteristics of the RM system equation.

Hypothesis III.b: it is possible to extend the TSK FLSs in a way that they become periodic and of continuous output, even for the 0-2π transitions of attitude Euler angle system inputs. Then for modelling multi-rotor flight dynamics each nonlinear component of a flight dynamics formulated by Euler Lagrange approach can be identified in a similar manner as stated in my previously described hypothesis III.a for RMs.

Hypothesis IV: system trajectories can be designed in harmony with the system dynamics and its actuator characteristics. Such trajectories are energy efficient as no oscillations are induced, and they are feasible, time optimal in terms that no trajectory exists with faster transients, such that the system can precisely track it with lesser energy consumption. These harmonic trajectories are continuous up to the required number of time derivatives, and they can be made bounded in their any number of time derivatives. For a realistic, feasible control input of multi-rotor UAV the designed path has to be such that the sixth time derivative of the body displacement function must be continuous and its fourth time derivative transient has to be feasible for the control actuator. For a realistic, feasible control input of direct brushless DC electric motor (BLDC) actuated systems (RMs, cranes, wheeled vehicles) the designed path has to be such that the fourth time derivative of the planned displacement must be continuous, while the planned body rotation must be such that the feasible body torque transients are proportional to the possible motor torque transients; equivalently the feasible second derivative of the body displacement has to be proportional to motor shaft feasible angular velocity.

Hypothesis V: for dynamic system GFS identifications the necessary training data set of collected samples along real trajectories can be reduced without significant loss in the quality of the identification result, while significantly improving the efficiency of the identification process.

Research Tools and Thesis Validation Methods

From the first introductory chapter it is obvious that my research is multidisciplinary, as such various research and test methods are necessary to test my hypothesis. As my goals are more general than finding a single specific method which is only applicable to UAV design, but are applicable to wide range of multidisciplinary field, I and not using a single UAV example to validate my theses. For each hypothesis I am also using a method well matched to the nature and specifics of the problem, so that my results are appropriately tested and presented in a general way.

1. Validating Quality of Multi-objective Search and Optimisation

The proposal is to be validated on well-studied, mathematically sound GA hard multi-objective benchmark problems like:

a) Simple Two Objective Optimisation Problem
b) Deceptive Multi-objective Optimisation Problem
c) Multi-modal Multi-objective Problem
d) Convex and Non-convex Pareto-optimal Fronts
e) Discontinuous Pareto-optimal Front
f) Biased Search Space
g) Generalisation of Two Objectives to Four, Eight and Sixteen Objectives

2. Validating Quality of Genetic Fuzzy System Function Approximation

The proposal is to be validated on well-studied, mathematically sound, versatile, difficult benchmark identification problems of high complexity like:

a) Predicting Future Values of Chaotic Time Series of Mackey and Glass

b) Identification of Gas Furnace Model of Box and Jenkins

c) Identification of Generalised Rastrigin Function

3. Validating Quality of Complex Nonlinear Dynamics System GFS Modelling

The proposal is to be validated on well-studied robot manipulator dynamics modelling and quadrotor flight dynamics modelling simulations.

4. Validating Quality of System Trajectory

The proposal is to be validated on well-studied 3D crane and quadrotor flight trajectory design.

5. Validating Quality of Genetic Fuzzy System Training Data Sets

The proposal is to be validated on well-studied quadrotor flight dynamics modelling simulations.

NEW SCIENTIFIC ACHIEVEMENTS

1 Vector Comparison Operators

New methods for comparing two GA objective vectors are introduced in my publication [s2]. Two proper strict inequality operators are formulated in my first thesis, and also an additional comparison method is proposed.

1.1 Quantity-dominance Vector Inequality Operator

To introduce the quantity-dominance definition for a minimisation problem, let's define a dominance relation $<_n(\boldsymbol{a}, \boldsymbol{b})$ (or briefly $\boldsymbol{a} <_n \boldsymbol{b}$) between two vectors of n elements: $\boldsymbol{a} = (a_i)$ and $\boldsymbol{b} = (b_i)$, for $i=1..n$, $n \in \mathbb{N}^+$, where each i^{th} element type has a well-defined scalar '**<**' (less than) strict partial order binary endorelation and also the equivalence relation '=' is defined.

Let's define a helper function $\#_{n<}(\boldsymbol{a}, \boldsymbol{b})$, which for vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ defines two values $(g_a, l_a) = \#_{n<}(\boldsymbol{a}, \boldsymbol{b})$, where $g_a, l_a \in \mathbb{N}_0$ and g_a is equal to the cardinality of set $G_{ab}=\{ a_i \mid b_i < a_i \}$, $i=1..n$; and l_a is equal to the cardinality of set $L_{ab}=\{ a_j \mid a_j < b_j \}$, $j=1..n$.

THESIS I.a - DEFINITION:

For a minimisation problem vector $\boldsymbol{a}$ **quantity-dominate**s vector $\boldsymbol{b}$, or briefly: $\boldsymbol{a} <_n \boldsymbol{b}$ if and only if $\boldsymbol{g_a} < \boldsymbol{l_a}$.

We can define a **measurement value** for $<_n(\boldsymbol{a}, \boldsymbol{b})$ as $\mathbf{d}_{<n}(\boldsymbol{a}, \boldsymbol{b}) = \boldsymbol{l_a} - \boldsymbol{g_a}$.

In my dissertation it is proven that the **quantity-dominance operator** $>_n(\boldsymbol{a},\boldsymbol{b})$ **(**$\boldsymbol{a} >_n \boldsymbol{b}$**) is a strict partial order binary endorelation**.

1.2 Quality–dominance Vector Inequality Operator

Let's define a dominance relation $<_q(\boldsymbol{a}, \boldsymbol{b})$ (or briefly $\boldsymbol{a} <_q \boldsymbol{b}$) between two vectors of n elements $\boldsymbol{a} = (a_i)$ and $\boldsymbol{b} = (b_i)$, for $i=1..n$, $n \in \mathbb{N}^+$, where each i^{th} element type has a well-defined scalar '<' (less than) strict partial order binary endorelation and also the equivalence relation '=' is defined.

Let's define a helper function $\#_{q<}(\boldsymbol{a}, \boldsymbol{b})$, which for vectors ***a*** and ***b*** defines two values $(g_a, l_a) = \#_{q<}(\boldsymbol{a}, \boldsymbol{b})$, where $g_a, l_a \in \mathbb{N}_0$ and g_a is equal to the cardinality of set $G_{ab}=\{ a_i \mid b_i < a_i \}$, $i=1..n$; and l_a is equal to the cardinality of set $L_{ab}=\{ a_j \mid a_j < b_j \}$, $j=1..n$.

THESIS I.b - DEFINITION:

For a minimisation problem vector ***a*** **quality-dominates** vector ***b***, or briefly: $\boldsymbol{a} <_q \boldsymbol{b}$ if $g_a < l_a$ or in case of $g_a = l_a$ ***a*** quality-dominates vector ***b*** if $\sum_i (a_i - b_i) < \sum_j (b_j - a_j)$, where *i* is such that $a_i \in G_{ab}$ and *j* is such that $a_j \in L_{ab}$.

We can define a **measurement value** for $<_q(\boldsymbol{a}, \boldsymbol{b})$ as

$$d_{<q}(\boldsymbol{a}, \boldsymbol{b}) = \begin{cases} l_a - g_a, & \text{for } g_a < l_a \\ \sum_j \frac{(b_j - a_j)}{l_a} - \sum_i \frac{(a_i - b_i)}{g_a}, & \text{for } g_a = l_a \end{cases}$$

, where *i* is such that $a_i \in G_{ab}$ and *j* is such that $a_j \in L_{ab}$.

In my dissertation it is proven that the **quality-dominance operator** $>_q(\boldsymbol{a},\boldsymbol{b})$ **(*a*** $>_q$ ***b*) is a strict partial order binary endorelation**.

1.3 Any–dominance Vector Comparison Method

Let's define a dominance relation $<_a(\boldsymbol{a}, \boldsymbol{b})$ (or briefly $\boldsymbol{a} <_a \boldsymbol{b}$) between two vectors of *n* elements $\boldsymbol{a} = (a_i)$ and $\boldsymbol{b} = (b_i)$, for $i=1..n$, $n \in \mathbb{N}^+$, where each i^{th} element type has a well-defined scalar '<' (less than) strict partial order binary endorelation and also the equivalence relation '=' is defined.

THESIS I.c - DEFINITION:

For a minimisation problem vector ***a*** **any-dominates** vector ***b***, or briefly: $\boldsymbol{a} <_a \boldsymbol{b}$ if and only if ($\boldsymbol{a} <_q \boldsymbol{b}$ OR $\boldsymbol{a} <_s \boldsymbol{b}$).

We can define a **measurement value** for $<_a(\boldsymbol{a}, \boldsymbol{b})$ as

$$d_{<a}(\boldsymbol{a}, \boldsymbol{b}) = \frac{l_a - g_a + \sum_i (b_i - a_i)}{2n},$$ where $i=1..n$.

This method is valid only if all scalar components a_i and b_i are in the same range (normalised to the closed interval of [0,1] for example). For GAs this normalisation can be simply achieved as we are investigating a finite number of objective vectors when determining the fitness of an individual in the population.

1.4 Non-dominance Measurement Based Ranking

Measurement based ranking in multi-objective GAs is a new possibility in rank assignment, which is made possible by the definition of measurements in my Thesis I.a, I.b, I.c. In analogy to MOGA – Block Type Non-dominance Ranking introduced in [23] we can calculate with all $\boldsymbol{b}_i^t$ individuals, by which the observed vector is dominated; but instead of the pure number of such vectors, I'm proposing to sum up the measurements of being dominated.

THESIS I.d - DEFINITION:

At generation t the **non-dominance measurement based rank** of the i^{th} individual $\boldsymbol{a}_i^t$ in a GA population, which is dominated by $\boldsymbol{b}_j^t$ individuals in the current population is the i^{th} individual current position; the individual's rank can be defined as:

- $rank_i(\boldsymbol{a}_i^t)$ = sum of the non-dominated comparison measurements for every other $\boldsymbol{b}_j^t$ individual of generation t in correlation to the i^{th} individual.

$rank(\boldsymbol{a}_i^t) = \sum_{j=1}^{n} \boldsymbol{d}_{<*}(\boldsymbol{b}_j^t, \boldsymbol{a}_i^t)$, where '*' can stand for any comparison method: {'s', 'P', 'n', 'q' or 'a'}.

1.5 Dominance Based Ranking

In analogy to MOGA – Block Type Non-dominance Ranking introduced in [23] we can calculate with all $\boldsymbol{b}_i^t$ individuals, but not those that dominate the observed vector, but with those, which are dominated by the observed vector.

THESIS I.e - DEFINITION:

At generation t the **dominance based rank** of the i^{th} individual $\boldsymbol{a}_i^t$ in a GA population is the count of all $\boldsymbol{b}_j^t$ individuals in the current population, which are dominated by $\boldsymbol{a}_i^t$ is the i^{th} individual current position; the individual's rank can be defined as:

- $rank_i(\boldsymbol{a}_i^t)$ = 1 + sum of the dominated $\boldsymbol{b}_j^t$ individuals of generation t in correlation to the i^{th} individual.

$rank(\boldsymbol{a}_i^t) = 1 + \#L_{ab} = 1 + \#\{ \boldsymbol{a}_i^t \mid \boldsymbol{a}_i^t <_* \boldsymbol{b}_j^t \}$, where '*' can stand for any comparison method: {'s', 'P', 'n', 'q' or 'a'}.

1.6 Dominance Measurement Based Ranking

Similarly to non-dominance measurement based ranking we can sum up the measurements of dominance for each dominated $\boldsymbol{b}_j^t$ individual in the current population t.

THESIS I.f - DEFINITION:

At generation t the **dominance measurement based rank** of the i^{th} individual $\boldsymbol{a}_i^t$ in a GA population, which dominates all $\boldsymbol{b}_j^t$ individuals in the current population is the i^{th} individual current position, the individual's rank can be defined as:

- $rank_i(\boldsymbol{a}_i^t)$ = sum of the dominated comparison measurements for every other $\boldsymbol{b}_j^t$ individual of generation t in correlation to the i^{th} individual.

$rank(\boldsymbol{a}_i^t) = \sum_{j=1}^{n} \boldsymbol{d}_{<*}(\boldsymbol{a}_i^t, \boldsymbol{b}_j^t)$, where '*' can stand for any comparison method: {'s', 'P', 'n', 'q' or 'a'}.

2 Minimalistic Parametrisation of Zadeh-type Fuzzy Partitions for Function Identification by Unconstrained Tuning

As described in [s3] the nature of Zadeh-formed MFs is such that simply making equal the last two parameters of the preceding MF to the first two parameters of the succeeding MF we easily form fuzzy partitions. This way a fuzzy partition of K MFs is defined by $2(K-1)+1$parameters. Let our input space be normalised ($x_{\min}$=0 and $x_{\max}$=1). If we do not want to allow any plateaux, parameter b_2 must be equal to b_3 in (25), thus the number of parameters for a fuzzy partition consisting of K pieces of Zadeh-type MFs is further reduced to the minimum of *(K-1)*.

If we take into consideration all of the constraints (26) we end up with a series of strictly ordered parameters:

$$b_1<b_2<\ldots<b_{K-1}. \tag{31}$$

Let us add two more constraints, which are possible as the input space is normalised:

$$0<b_1 \text{ and } b_{K-1}<1. \tag{32}$$

Let us define the first MF to be:

$$mfz(x,0,b_1), \tag{33}$$

and the K^{th}, the last one, to be:

$$mfs(x,b_{K-1},1). \tag{34}$$

Let us define the general intermediate k^{th} MF to be:

$$mf\pi(x,b_{k-1},b_k,b_k,b_{k+1}) \tag{35}$$

for $k = 2, \ldots,(K-1)$. This way the ordered series of *(K-1)* parameters (31) together with border conditions (32) are the minimal number of parameters to define a fuzzy-partition of Zadeh-formed MFs, which can represent any such partition.

This minimal number of nonlinear parameters is a very important issue for optimisation as over parameterised systems are hard to optimise. The only problem now remains to be that when we are tuning these interdependent b_k nonlinear parameters of a FLS having an n dimensional input space, we must comply with $\sum_{i=1}^{n} K_i$ pieces of hard constraints. Although there are a number of constrained optimisation methods it is obvious that an unconstrained optimisation method would be more efficient. My proposal is to represent the b_k parameters in a different manner.

THESIS II - DEFINITION:

For **a minimal independent parametrisation of Zadeh-type MF based fuzzy partitions, that can be optimised without any constraints**, let us consider K pieces of rational, positive or zero parameters as:

$$a_\kappa \in R_0^+, \kappa = 1,\ldots,K. \tag{36}$$

Let us form the b_k nonlinear parameters of Zadeh-type MFs forming fuzzy-partitions for a FLS as:

$$b_k = \sum_{j=1}^{k} a_j \Big/ \sum_{\kappa=1}^{K} a_\kappa, \tag{37}$$

then for every $k = 1, \ldots, K$ all the constraints (31) and (32) are automatically fulfilled for every b_k from (37) without any further restrictions on a_κ.

3 Genetic Fuzzy System Grey-box Modelling of Complex Dynamics Systems

As described in [s4], [s5], [s10], [s11] my proposed identification method for a general Robot Manipulator (RM) Dynamics equation (39) identification is to use Zadeh-formed membership functions (MFs) as in equation (25) for antecedents as in equation (23) in a Takagi-Sugeno-Kang (TSK) type FLS having n inputs and 1 output as defined in (27). MF nonlinear parameters are represented as in equation (37). Centrifugal and Coriolis components are calculated from the Inertia component as in equation (41).

This chapter relays heavily on many complex equations described in chapters 3 and 4.1 – I have repeated here the bare equations to support a brief background overview for my Thesis III.a:

$$\sum_{j=1}^{p}\left(\boldsymbol{D}_{ij}(\boldsymbol{q})\cdot\ddot{\boldsymbol{q}}_j\right)+\sum_{j=1}^{p}\sum_{k=1}^{p}\left(\dot{\boldsymbol{q}}_{\boldsymbol{j}}\cdot\boldsymbol{D}_{ijk}(\boldsymbol{q})\cdot\dot{\boldsymbol{q}}_k\right)+\boldsymbol{D}_i(\boldsymbol{q})+f_i=\tau_i \qquad (39)$$

$$f(\boldsymbol{x})=\sum_{l=1}^{M}\omega_l(\boldsymbol{x})\cdot y_l(\boldsymbol{x}). \qquad (27)$$

$$y_l=\sum_{j=1}^{n}c_{l(j)}\cdot x_j+c_{l(0)} \qquad (24)$$

$$\omega_l(\boldsymbol{x})=\prod_{i=1}^{n}\mu_{F_l(i)}(x_i) \qquad (23)$$

where each MF $\mu_k(x,\boldsymbol{b})$ is chosen from equations (25) in such a manner that for every input *x* it holds that $\sum_{k=1}^{K}\mu_k\,(x,\ b)=1$, the MFs are said to form a fuzzy-partition. A common partition scheme is to start with *mfz*, and finish with *mfs*, while having arbitrary number of $mf\pi$s in-between.

$$mfz(x,b_1,b_2)=\begin{cases}1 & x\le b_1\\ 1-2((x-b_1)/(b_2-b_1))^2 & b_1<x\le \tfrac{1}{2}(b_2+b_1)\\ 2((b_2-x)/(b_2-b_1))^2 & \tfrac{1}{2}(b_2+b_1)<x\le b_2\\ 0 & x>b_2\end{cases}$$

$$mfs(x,b_1,b_2)=1-mfz(x,b_1,b_2)$$

$$mf\pi(x,b_1,b_2,b_3,b_4)=\begin{cases}mfs(x,b_1,b_2) & x\le b_2\\ 1 & b_2<x\le b_3\\ mfz(x,b_3,b_4) & x>b_3\end{cases}, \qquad (25)$$

$$b_k=\sum_{j=1}^{k}a_j\Big/\sum_{\kappa=1}^{K}a_\kappa, \qquad (37)$$

$$D_{ijk}=\frac{1}{2}\left(\frac{\partial D_{ij}}{\partial q_k}+\frac{\partial D_{ik}}{\partial q_j}-\frac{\partial D_{jk}}{\partial q_i}\right),D_{ijk}=D_{ikj},D_{kij}=-D_{jik},D_{kjk}=0,\forall i,k\ge j, \qquad (41)$$

THESIS III.a - DEFINITION:

(Robot manipulator) **Complex system dynamics** equation (39) **can be precisely identified by approximating its $\boldsymbol{D}_{ij}$ inertia, $\boldsymbol{D}_i$ gravity and f_i friction components with FLSs** as:

$\boldsymbol{D}_i(\boldsymbol{q})$ and $\boldsymbol{D}_{ij}(\boldsymbol{q}) = \sum_{l=1}^{M}\left(\left(\prod_{i=1}^{n}\mu_{F_l(i)}(q_i, \boldsymbol{b_l})\right)\cdot\left(\sum_{j=1}^{n}c_{l(j)}\cdot q_j + c_{l(0)}\right)\right)$, and

$$f_i(q_i) = \sum_{f=1}^{F}\left(\mu_{F_f(i)}\left(q_i, \boldsymbol{b}_f\right)\cdot\left(c_{f(i)}\cdot q_i + c_{f(0)}\right)\right)$$

where n is the number of position state variables (number of RM joints); M and F is the designed number of FLS rules; q_i is the i[th] position state variable (RM joint position); $c_{l(j)}$ and $c_{f(i)}$ are the linear parameters to be identified; $\boldsymbol{b_l}$ and $\boldsymbol{b_f}$ are vectors of nonlinear parameters of Zadeh MF formed fuzzy partitions as in equation (25). Components of $\boldsymbol{b_l}, \boldsymbol{b_f}$ are formed as equation (36), (37) in my Thesis II:

$$b_{*,k} = \sum_{j=1}^{k} a_{*,j} \Big/ \sum_{\kappa=1}^{K} a_{*,\kappa}\ ,\ a_{l,\kappa} \in R_0^+, \kappa = 1,...,K_* \tag{37}$$

where K_* corresponds to K_l and K_f, the designed number of input membership functions of fuzzy partition for $\boldsymbol{D}_{ij}$, $\boldsymbol{D}_i$ and f_i FLS antecedents.

Dynamics equation (39) $\boldsymbol{D}_{ijk}$ components are to be expressed from the FLS form of $\boldsymbol{D}_{ij}$ inertia components by applying Christoffel symbols as:

$$D_{ijk} = \frac{1}{2}\left(\frac{\partial D_{ij}}{\partial q_k} + \frac{\partial D_{ik}}{\partial q_j} - \frac{\partial D_{jk}}{\partial q_i}\right), D_{ijk} = D_{ikj}, D_{kij} = -D_{jik}, D_{kjk} = 0, \forall i,k \geq j, \tag{41}$$

$$D_{ij} = D_{ji}, D_{ijk} = D_{ikj}, D_{kij} = -D_{jik}, D_{kjk} = 0,\ \forall i,k \geq j \tag{46}$$

For $a, b, c\ \in \{i, j, k\}$ we have $\frac{\partial D_{ab}(\boldsymbol{q})}{\partial q_c} = \sum_{l=1}^{M}\left(\frac{\partial\left(\prod_{i=1}^{n}\mu_{F_l(i)}(q_i,\boldsymbol{b_l})\right)}{\partial q_c}\cdot\left(\sum_{j=1}^{n}c_{l(j)}\cdot q_j + c_{l(0)}\right) + \left(\prod_{i=1}^{n}\mu_{F_l(i)}(q_i, \boldsymbol{b_l}\right)\frac{\partial\left(\sum_{j=1}^{n}c_{l(j)}\cdot q_j + c_{l(0)}\right)}{\partial q_c}\right)$.

The (RM) system dynamics equation (39) can now be stated as: $\tau_i = \sum_{j=1}^{N}A_{ij}(\ddot{\boldsymbol{q}}, \dot{\boldsymbol{q}}, \boldsymbol{q}, a_{ijl\kappa})\cdot c_{il(j)}$, and the complete body dynamics is now of the form: $\boldsymbol{\tau} = \boldsymbol{A}(\ddot{\boldsymbol{q}}, \dot{\boldsymbol{q}}, \boldsymbol{q}, \boldsymbol{a_\kappa})\cdot\boldsymbol{c}$, where τ_i is the i[th] body torque; $\boldsymbol{q}$ is the body position system state, $\dot{\boldsymbol{q}}$ is its first time derivative and $\ddot{\boldsymbol{q}}$ is its second time derivative; A_{ij} is a very complex nonlinear equation to write down, while relatively simply expressed by the stated FLS identification procedure.

Linear system parameters c_* – components of vector $\boldsymbol{c}$ are to be calculated by SVD decomposition based LS optimal method as: $\boldsymbol{c} = \boldsymbol{VS^{-1}U^T}\cdot\boldsymbol{\tau}$ for SVD decomposition of $\boldsymbol{A}(\ddot{\boldsymbol{q}}, \dot{\boldsymbol{q}}, \boldsymbol{q}, \boldsymbol{a_\kappa}) = \boldsymbol{USV^T}$.

Nonlinear system parameters $a_{*,\kappa}$ – components of vector $\boldsymbol{a_\kappa}$ are to be identified with a global stochastic search method, like GAs of my Thesis I, and fine-tuned by a gradient descent method.

4 Continuous Periodic Fuzzy Logic Systems

As described in [s6], [s13], [s14] my proposal is to transform the general FLS equation (27) to form a continuous periodic FLS (cpFLS). Such cpFLSs are ready to be used for modelling systems which are inherently continuous and periodic, for example the orientation angle input based torque function of a multi-rotor dynamics in equation (43).

For physical systems in the Euclidian space orientation angles are naturally defined on the [0, 2π) interval. Any angular value α below *0* or above *2π* is equivalent to a value $\beta=\alpha\pm2k\pi$, where k is such an ordinary number that $\beta\in[0,2\pi)$. For orientation angles selection of the origin is arbitrary and transition between two orientation angles is continuous and smooth, as in having a continuous first derivative. As orientation angle of 2π is equivalent to angle 0 the transition from 2π-ε to 0+ε also has to be continuous.

For FLSs defined by equation (27) we can make the input space continuous and periodic over the *[0, 2π)* interval by applying a simple piecewise linear "seesaw" function transformation, by which we make sure that there is no discontinuity between angular cpFLS inputs of any two values, we simply force critical *2π-ε* input values to become equal to *0+ε* for all $\varepsilon < \pi/2$. This step is needed to ensure the output space $y_l(\boldsymbol{q})$ consequence part can become continuous over the $\boldsymbol{q}$ ∈*[0, 2π)* input space even for full circle rotations.

We also have to make the antecedent fuzzy partition "circular" by combining the first μ_z and the last μ_s MF of the partition as defined in equation (25) into a single virtual μ_π MF to be substituted into equation (23), so that fuzzy rules applied to the first z-MF equally apply to the last s-MF. We achieve this by making all the linear parameters of the last rule for each fuzzy partition in equation (27) equivalent to the first rule of the same partition as $c_{jK_i} = c_{jK_1}$, where *n* is the number of cpFLS inputs, and each input is covered by a fuzzy partition of K_i MFs for *i=1..n*.

By this procedure we have ensured to have a continuous periodic fuzzy system (cpFLS) such that for $\forall \boldsymbol{q} \in \mathbb{R}^n, \forall k \in \mathbb{Z}$ and any arbitrary small $\boldsymbol{\varepsilon}$ there is a similarly small $\mu(\boldsymbol{\varepsilon})$ for which we have:

$$cpFLS(\boldsymbol{q} \pm 2k\pi) = cpFLS(\boldsymbol{q}), \; cpFLS(\boldsymbol{q} \pm \boldsymbol{\varepsilon}) = cpFLS(\boldsymbol{q}) \pm \mu(\boldsymbol{\varepsilon}), \tag{48}$$

THESIS III.b - DEFINITION:

All FLSs that for antecedent use Zadeh-type MF based fuzzy partitions, whose last (*smf*) and first MF (*zmf*) can form a single continuous MF (*πmf*), and the consequent part of rules is a constant (like in Mamdani FLS) or a continuous function of the input signal (like a TSK FLS), can be made **continuous and periodic fuzzy logic system (cpFLS)** as in equation (48) by:

1. applying equation (47) as a preliminary transformation to the input signal

2. making all parameters c_{jK_i} of fuzzy rule consequents whose premise includes the *smf* identical to c_{jK_1} parameters of rule consequents for the matching *zmf* of the same input.

5 Genetic Fuzzy System Grey-box Modelling of Multi-rotor Flight Dynamics

As described in [s6], [s13], [s14] my proposal is to use continuous periodic FLS (cpFLS) for modelling systems which are inherently continuous and periodic, for example the orientation angle input based torque function of a multi-rotor dynamics in equation (43).

THESIS III.c - DEFINITION:

(Multi-rotor) **flight dynamics can be precisely identified by continuous and periodic fuzzy logic systems** by taking system components for cpFLSs (my Thesis III.b) as in equations (39), (46), (50), (51), (52) for $\boldsymbol{q}$ = *(ϕ, θ, ψ),* by applying the identification method as equation (53), which is detailed in my Thesis III.a as:

$$\sum_{j=1}^{p}\left(\boldsymbol{D}_{ij}(\boldsymbol{q}) \cdot \ddot{\boldsymbol{q}}_j\right) + \sum_{j=1}^{p}\sum_{k=1}^{p}\left(\dot{\boldsymbol{q}}_{\boldsymbol{j}} \cdot \boldsymbol{D}_{ijk}(\boldsymbol{q}) \cdot \dot{\boldsymbol{q}}_k\right) + \boldsymbol{D}_i(\boldsymbol{q}) + f_i = \tau_i, \tag{39}$$

$$D_{13}(\theta) = f_1(\theta), D_{22}(\phi) = f_2(\phi), D_{23}(\phi,\theta) = f_3(\phi,\theta), D_{33}(\phi,\theta) = f_4(\phi,\theta), \tag{50}$$

$$D_{11} = I_{xx}, D_{12} = 0, D_{21} = D_{12}, D_{31} = D_{13}, D_{32} = D_{23}, \tag{51}$$

$$D_{122} = -\frac{1}{2}\frac{\delta D_{22}}{\delta \phi}, D_{123} = \frac{1}{2}\left(\frac{\delta D_{13}}{\delta \theta} - \frac{\delta D_{23}}{\delta \phi}\right), D_{322} = \frac{\delta D_{23}}{\delta \theta},$$
$$D_{133} = -\frac{1}{2}\frac{\delta D_{33}}{\delta \phi}, D_{223} = -\frac{1}{2}\frac{\delta D_{33}}{\delta \theta}, D_{312} = \frac{1}{2}\left(\frac{\delta D_{23}}{\delta \phi} + \frac{\delta D_{13}}{\delta \theta}\right) \tag{52}$$

$$D_{ij} = D_{ji}, D_{ijk} = D_{ikj}, D_{kij} = -D_{jik}, D_{kjk} = 0, \; \forall i,k \geq j, \tag{46}$$

$$(\mathbb{J}^*(\boldsymbol{q}, \boldsymbol{a_\kappa}) \cdot \ddot{\boldsymbol{q}} + \mathbb{C}^*(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{a}_\kappa) \cdot \dot{\boldsymbol{q}}) \cdot \boldsymbol{c} = \boldsymbol{A}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}, \boldsymbol{a}_\kappa) \cdot \boldsymbol{c} = \boldsymbol{\tau}, \tag{53}$$

For such a flight dynamics system model the minimal number of $\boldsymbol{a_\kappa}$ nonlinear parameters is 24 and the number of $\boldsymbol{c}$ linear parameters is 113 to achieve a good quality model, when the f_i friction components are neglected; $\boldsymbol{D_i}$ gravity components for a free flying object are non-existent.

In analogy to my Thesis III.a, linear flight dynamics system parameters c_* – components of vector $\boldsymbol{c}$ are to be calculated by SVD decomposition based LS optimal method as: $\boldsymbol{c} = \boldsymbol{VS^{-1}U^T} \cdot \boldsymbol{\tau}$ for SVD decomposition of $\boldsymbol{A}(\ddot{\boldsymbol{q}}, \dot{\boldsymbol{q}}, \boldsymbol{q}, \boldsymbol{a_\kappa}) = \boldsymbol{USV^T}$.

Nonlinear system parameters $a_{*,\kappa}$ – components of vector $\boldsymbol{a_\kappa}$ are to be identified with a global stochastic search method, like GAs of my Thesis I, and fine-tuned by a gradient descent method.

6 Feasible Optimal Harmonic Trajectories of Bounded, Smooth Time Derivatives

In [37] the rotor blade velocity is considered as an arbitrary control input. As 7^{th} order minimum-snap polynomial trajectories are discontinuous in displacement crackle, fifth derivative of displacement, my claim in [s14], [s15] is that this is still a sub-optimal approach; again: the rotor blade velocity is not an arbitrary theoretical control signal, but a real, electro-mechanical physical system, subject to aero dynamical load conditions.

The goal of this paper is to present a new method for flexible and efficient real-time direct path parametrisation, which is capable of generating physically feasible, time-and energy optimal, bounded, continuous trajectories with minimal induced oscillations; a method even usable for autonomous navigation. The notion of time and energy optimality is not used in mathematics theory manner, but in real life, physically feasible engineering manner [s14], [s15].

The process of finding optimal trajectories is in this paper focused on finding the appropriate parametrisation for the path vector function ***f(t)***, given the pre-defined feasibility limits on the displacement time derivatives, in conjunction with the effects of the path curvature.

The defined boundary conditions of the trajectory have to be satisfied. The defined limits on maximum values for arbitrary time derivatives of the displacement have to be obeyed.

Continuity and smoothness of every trajectory component has to be ensured up to the predetermined order: six times smooth in case of multi-rotors, four times smooth in case of cranes and RMs.

As described in [s14] and [s15], to have realistic, feasible torques along a trajectory, which are efficiently controllable without chattering, we need smooth torque changes. For indirect rotor-blade propulsion systems (ships, multi-rotors) we have the propulsion motor force or torque $\boldsymbol{M_M}(t) \approx const \cdot \boldsymbol{\omega}(t)^2$ proportional to the square of the rotor angular velocity. The applied mechanical force or torque $\boldsymbol{M_B}(t) \approx m * \ddot{\boldsymbol{\mu}}(t)^2$ excreted onto the body is proportional with the second derivative of the linear position or rotation angle $\ddot{\boldsymbol{\mu}}(t)$ of the body. As the body is driven by a rotor blade, $\boldsymbol{\omega}(t)$ is proportional to $\ddot{\boldsymbol{\mu}}$, the body angular acceleration.

In reality no discontinuities can physically occur, not even in third time derivatives of a displacement neither for the controlled system, nor for the control actuator.

For a realistic, feasible control input of direct BLDC actuated systems (RMs, cranes, wheeled vehicles) the designed path has to be such that the planned snap ($\xi^{(4)}$) must be continuous and proportional to the third derivative of the motor shaft rotational displacement . Ultimately for a feasible trajectory for the body rotation we must obey that the feasible body torque transients are proportional to the possible motor torque transients; equivalently the feasible second derivative of the body displacement $\xi^{(2)}(t)$ has to be proportional to motor shaft possible $\omega(t)$. On top of the allowed trajectory transient behaviour there are requirements on its smoothness as well. To be able to optimally control an electric motor with either $\boldsymbol{v_e}(t)$ or $\frac{\boldsymbol{di_e}}{\boldsymbol{dt}}(t)$, the $\frac{\boldsymbol{d^2\omega}}{\boldsymbol{dt^2}}(t)$ signal has to be continuous; equivalently $\xi^{(4)}$(t), snap, the fourth time derivative of body displacement has to be continuous.

Dependency of multirotor torque and rotor blade angular velocity on the continuity of the pop function can be also demonstrated by simply calculating and plotting these system values for an artificially created step function-like trajectory pop [s14] – it is well notable that any discontinuity in the trajectory pop will result in a discontinuity in the time derivative of the required rotor angular velocity, which we have already concluded to be a physical not feasible requirement. The most important system variable time signals of such infeasible discontinuous trajectories are presented in Appendix II.

THESIS IV.a - DEFINITION:

For a **realistic, feasible general system trajectory** one must design realistic, feasible control system inputs. For direct BLDC actuated systems (RMs, cranes, wheeled vehicles) the designed path has to be such that the planned body displacement fourth time derivative, the snap ($\xi^{(4)}$) must be continuous and proportional to the third derivative of the motor shaft rotational displacement as in equation (62).

7 Feasible Optimal Harmonic Multi-rotor Flight Trajectories

For a realistic, feasible control input of multi-rotor UAVs, we must not only consider equation (54), but also (55) and (58), so the designed UAV path has to be such that the displacement pop ($\boldsymbol{\xi}^{(6)}$) must be continuous and the body snap transient has to be feasible by a BLDC: $\boldsymbol{\xi}(t)^{(4)} \sim \boldsymbol{\omega}(t)$.

As described in [s6], [s7], [s13], [s14] to have realistic, feasible torques along a trajectory, which are efficiently controllable without chattering, we need smooth torque changes. The term (n times) smooth is used as in being equivalent to having continuous (n^{th}) time derivative.

THESIS IV.b - DEFINITION:

For a **realistic, feasible multi-rotor trajectory** one must design realistic, feasible control system inputs, such that the planned body displacement sixth time derivative, the pop ($\xi^{(6)}$) must be continuous and proportional to the third derivative of the motor shaft rotational displacement as in equation (62).

THESIS IV.c - DEFINITION:

A **realistic, feasible multi-rotor trajectory parametrisation** of continuous body displacement sixth time derivative pop ($\xi^{(6)}$), such that the snap ($\xi^{(4)}$) is proportional to the motor shaft rotational velocity as in equation (62) can be designed by selecting:

$$\boldsymbol{p_t}(t) = \boldsymbol{\xi_t^{(6)}}(t) = G \cdot \frac{2\pi}{P} \sin\left(\frac{2\pi}{P} t\right). \tag{65}$$

$$\boldsymbol{c_t}(t) = \boldsymbol{\xi_t^{(5)}}(t) = G \cdot \left(1 - cos\left(\frac{2\pi}{P} t\right)\right), \tag{64}$$

where P is either measured, as in equation (62), or calculated based on equation (63)

$$\boldsymbol{\omega_t}(t) = \frac{\omega_{stat}}{2}\left(1 + tanh\left(\frac{\pi}{P}\left(t - \frac{P}{4}\right)\right)\right), \tag{62}$$

$$P = \frac{2\pi}{A} = 2\pi\sqrt{\left(\frac{C^2}{B^2} + \frac{BD}{B^2}E\right)^{-1}} = \frac{2\pi L_e(J_M+J_R)}{\sqrt{(R_e(J_M+J_R)+L_e\gamma_M)^2+EL_e(J_M+J_R)L_eK_d}} \tag{63}$$

8 Feasible Optimal Harmonic 3D Overhead Crane Trajectories

Compared to the analysed multi-rotor dynamics, a 3D overhead crane model and a RM dynamics model are of the same basic format as equation (39), these systems are more simple as the position of the payload or end effector is directly linked to the position of the actuator rotor shaft – there is no intermediate transfer function like equation (54) for a multi-rotor. This fact predicts that cranes and RMs are not sensitive to discontinuities in the trajectory pop or crackle, only the snap has to be continuous.

THESIS IV.d - DEFINITION:

When the trajectory is planned as in my Thesis IV.c with parameter P matching the system transient behaviour, the trajectory is harmonic.

For a **harmonic, realistic, feasible multi-rotor trajectories induce no system oscillations.**

9 Singular Value Decomposition Based Genetic Fuzzy System Training Data Set Reduction

As described in [s6] when identifying a system, we have to design a sufficiently exciting trajectory, which will properly expose all singular values of the (linear) system. For a stable equation solution for linear parameters it is needed to have all singular values higher than one.

For solving a linear system of equations it is recommended to use an SVD-based decomposition method before calculating the inverse matrix as for equation (53), but calculating SVD decomposition for large matrices is very processor and memory demanding task, which increases exponentially with the data set size.

Data samples collected along sufficiently exciting trajectories tend to be oversized, thus redundant. In [s12] and [s16] it is shown for a robotic manipulator dynamic model identification, that by using only a reduced number of training data points the same quality of system identification can be reached as with the full set, given that the reduced set is representative enough of the full set, which is equivalent to having a similarly low condition number.

The FLS training in this case is finding the proper $\boldsymbol{b}$ vector of the nonlinear MF parameters, and finding the $\boldsymbol{c}$ vector of linear consequence parameters. For optimising $\boldsymbol{b}$ one can us a method as described in my thesis II. For LS optimal $\boldsymbol{c}$ vector one can use the SVD transformation property as $\boldsymbol{c} = \boldsymbol{V}\boldsymbol{S}^{-1}\boldsymbol{U}^{\boldsymbol{T}} \cdot \boldsymbol{F}_{full}(\boldsymbol{x})$ for the SVD decomposition of $\boldsymbol{A}_{full} = [\boldsymbol{A}(\boldsymbol{x_i}, \boldsymbol{b})] = \boldsymbol{USV^T}$, where $\boldsymbol{F}_{full}(\boldsymbol{x}) = [f(\boldsymbol{x_i})]$ is the vector of the traning data results $f(\boldsymbol{x_i})$**,** for the input series data $\boldsymbol{x_i}$, *i=1,..,N;* for *N* being the number of traing data inputs.

The proposal of this paper is to apply a selection algorithm to $[\boldsymbol{x_i}]$ and thus to the $\boldsymbol{F}_{full}(\boldsymbol{x})$ training data set, such that we can determine an arbitrary quality / size balanced training data set $[\boldsymbol{x_j}]$ and thus $\boldsymbol{F}_{red} = \left[f\left(\boldsymbol{x_j}\right)\right]$ for FLS based dynamic model identifications.

THESIS V - DEFINITION:

Without compromising the identification quality it is possible to reduce an oversized training data set *F(x)* in a manner that we extract only samples $\boldsymbol{x_j}$ such that the selected input-output training data pairs $\boldsymbol{F}_{red} = \left[f\left(\boldsymbol{x_j}\right)\right]$ maximise the condition number decrease of the $\boldsymbol{A}_{red} = [\boldsymbol{A}(\boldsymbol{x_j}, \boldsymbol{b})]$ of the FLS antecedent matrix.

SUMMARY CONCLUSIONS

Results

1. *New Vector Comparison Operators*

This paper presents a new vector comparison relation operator, and its extensions that can be used for creating a measurement based new multi-objective ranking operator, which can be the bases for an efficient new multi-objective GA. Also a measurement function is defined for Pareto-dominance. A general measurement based ranking method is proposed. Also a modification of fitness sharing is presented. Numerous multi-objective GA types are evaluated for their performance on GA hard functions.

Each tested GA, no matter which ranking method is used, efficiently finds the close proximity of the true Pareto-front. The proposed new dominance based ranking methods DO and DM both outperform all other tested ranking methods by 20% when it comes to the number of generation evaluations required for convergence, and they also outperform the others by 5-10% when it comes to the number of non-dominated individuals found in the final generation.

Each tested GA, no matter which vector comparison method is used, efficiently finds the proximity of the true Pareto-front. The new vector comparison methods (A, N, Q) outperform the Pareto comparison by 5-15% when it comes to the number of generation evaluations required for convergence, and they also outperform the others by 5-15% when it comes to the number of non-dominated individuals found in the final generation.

2. *New Minimalistic Parametrisation of Zadeh-type Fuzzy Partitions for Function Identification by Unconstrained Tuning*

This paper presents a novel method that simplifies the $\boldsymbol{b_i}$ non-linear parameter optimisation of TSK FLSs based on fuzzy partitions for antecedent MFs like equation (27) that is suitable for unconstrained stochastic and gradient descent based non-linear optimisation, while preserving all the required constraints and properties. All linear parameters of equation (24) are determined by SVD based robust LS method.

The proposed identification method is capable of highly efficient off-line precise identification, and also real-time adaptive fine tuning of fuzzy systems for function approximation or system identification purposes. Furthermore, the proposed minimalistic parameterisation of Zadeh-formed MFs makes it possible to use unconstrained optimisation methods while the initial ordering of MFs and the fuzzy-partitioning properties are preserved.

The presented simple uniform partition based fuzzy precedent definition with SVD-based linear antecedent calculation is a very fast, good enough uniform function approximation technique. The application of my proposed precedent parameter representation enables the application of any numerically efficient unconstrained tuning of the fuzzy system. Applying a gradient-descent like method further improves the identification quality; at a cost of some extra computation effort (usually 15 iterations are satisfactory). Applying an initial efficient GA search for the global optima neighbourhood of the precedent parameters, combined with gradient-based fine toning and SVD-based antecedent parameter calculations result in extremely precise function identifications; at a cost of further extra computation effort (usually <15 generations are needed for a population proportional to the complexity of the problem, proportional to the dimension of the search space and the number of objectives).

This very efficient and minimalistic parametrisation of uniform function approximation fuzzy systems is the starting point of building complex, robust fuzzy system models, which can cope with real life data uncertainties such as the unpredictable aerial environment of an UAV.

3. *New Genetic Fuzzy System Grey-box Modelling of Complex Dynamics Systems*

This paper presents a new method that identifies the RM dynamics through finding the $\boldsymbol{D_{ij}}$ nonlinear functions of equation (39) as TSK FLSs, while calculating $\boldsymbol{D_{ijk}}$ nonlinear functions as in equation (41). All linear parameters of the system are determined by SVD based robust LS method. Nonlinear parameters are evolved by multi-objective GA and fine-tuned by gradient descent method.

This paper presents a new method that identifies the multi-rotor flight dynamics equation (44) $\boldsymbol{D_{ij}}$ components by specially constructed continuous and periodic TSK FLSs, while calculating the $\boldsymbol{D_{ijk}}$ nonlinear functions as in equation (45). All linear parameters of the system are determined by SVD based robust LS method. Nonlinear parameters are evolved by multi-objective GA and fine-tuned by gradient descent method.

The proposed identification method is capable of forming and fine-tuning a soft computing, fuzzy system based dynamic model for a robot manipulator. The number of nonlinear parameters can be kept to minimal and optimised by evolutionary and gradient based methods, too. The value of the linear parameters can be determined by a least squares method. After an initial evaluation the complete identification method is capable of running on-line with a control algorithm if we use an on-line iterative least squares method for the linear parameters [57], while from the background a hybrid evolutionary and gradient based method periodically updates the nonlinear parameters.

The relative value of the maximal error is well within the tolerance level of a model based control algorithms [80]. Parameters identified by this method can be considered as real physical values, in contrast to previous results where some negative numbers appeared for inertia terms.

The proposed identification method is capable of forming and fine-tuning a soft computing, fuzzy system based dynamic model for quadrotors. The quality of identification with the relative torque error being uniformly <10% is suitable for application in model based control algorithms; the torque error is presented in Figure 5. Such good quality UAV flight dynamics models are the prerequisites for quality model based flight control systems.

4. *New Feasible Optimal Harmonic Trajectories of Bounded, Smooth Time Derivatives*

This paper presents a novel harmonic path construction real-time direct algorithm for generating physically feasible, time-and energy optimal, bounded, continuous trajectories that can reach any target displacement with a known minimal error. These trajectories can be designed to arbitrary smoothness – depending on system requirements; they are to be designed smooth up to the 5^{th} time derivative of displacement for multi-rotor UAV trajectories. The term (n times) smooth is used as in being equivalent to having continuous (n^{th}) time derivative. The requirement for feasible trajectories of having minimum 5 times smooth displacement functions in case of UAV is proven. Effects of trajectory discontinuities on system state oscillations are studied in details. It is proven that the proposed harmonic trajectories of appropriate smoothness (defined by the system and control actuator dynamics) do not generate system state oscillations.

The proposed trajectory design method is capable of forming bounded, smooth, energy efficient and time optimal trajectories with a single pass algorithm using closed formulas. The design method is defined and validated on an example for a multi-rotor UAV path planning, where a single parameter controls the trajectory dynamics, as presented in Figure 32.

Dynamic transient properties and energy efficiency of the trajectory can be tuned with a single parameter, but the feasibility of torque transients must not be dismissed along this optimization. The resulting trajectory is always the time optimal solution, which complies with all defined limits.

5. *New Singular Value Decomposition Based Genetic Fuzzy System Training Data Set Reduction*

This paper presents a novel method that reduces the necessary training data set size for fuzzy identification of complex dynamic systems. The method is based on finding the minimal subset of the training data, which most efficiently minimises the corresponding condition number of the linear system subject to SVD decomposition when identifying the optimal linear parameters of the system.

The proposed GFS training data set reduction method, while maintaining the quality of the identification process, is capable of significantly reducing the number of necessary training data points, and thus significantly increases the identification process performance. The method is defined and validated on quadrotor dynamic model identification with GFS, where less than 20% of data points give more than 80% of contribution to the system condition number. A typical rate of condition number change for the most significant 25% of data points is presented in Figure 58.

The training data set reduction to 1/20th of the full set significantly increases the identification process speed, while the proposed reduction method ensures that the identification result quality does not deteriorate to below a pre-defined minimum precision level.

This method implicitly provides information on the quality of the training data set. The condition number is of acceptable magnitude only for full rank matrices. Rank deficiency in case of the proposed fuzzy identification methods means that there is no sufficient data to meaningfully define consequent values for every rule; thus when using such model for control purposes we cannot achieve uniform stability – a model built on a rank deficient fuzzy system is not stable for the complete operational space, even if the antecedent fuzzy partition uniformly cover the complete input space.

Application Possibilities of Results

The main goal of this work is to create new and improve existing tools, by which the complete autonomy with obstacle avoidance of UAV navigation can be enhanced.

My first thesis group gives a set of new tools for evolving, searching near-optimal parameters of complex systems such as fuzzy models of system dynamics as in navigation dynamics of UAV.

My second thesis group gives a new tool for minimalistic representation of fuzzy model parameters and their unconstrained tuning for precise function approximation in modelling complex system dynamics as in navigation dynamics of UAV.

My third thesis group gives a new tool for efficient complete fuzzy modelling of continuous and periodic complex nonlinear dynamics systems as in navigation dynamics of UAV.

My fourth thesis group gives a new tool for feasible optimal trajectory design, which can real-time generate trajectory parametrisations while obeying all the kinematic constraints, such as parametrisation for any geometric UAV path with velocity and acceleration constraints.

My fifth thesis group gives a new tool for efficient genetic fuzzy modelling by reducing the training data set to minimum, while guaranteeing the prescribed quality of the solution.

Along my results to improve the autonomy of UAV navigation I propose to:

- start from a flying UAV, keep the high level strategic way-point selection algorithm

- for planning the exact feasible optimal trajectory between two way-points use my method described in thesis 4

- at the initial stage use the existing control mechanism to track such feasible optimal trajectories and collect measurement signals datasets consisting of at least 3D position, 3D orientation data paired to exact (4 in case of quadrotor) motor rotation velocities for each time sample; if sensors can provide, further data can be collected such as position and orientation velocities and accelerations, motor currents or voltages

- for minimising the training data set size use my method described in thesis 5

- using my methods described in thesis 1 and 2 design a fuzzy system structure as described in thesis 3 to precisely model the UAV flight dynamics along the reduced trading data set

- replace the UAV control system to a back stepping (computed torque) controller, which uses a fuzzy reference model obtained in the previous step

A further improvement possibility exist in adapting the computed torque control algorithm in a manner that it does not apply a simple PID action to the decoupled double integrators, but instead actually calculates a feasible optimal harmonic micro-trajectory which, when super-positioned to the original trajectory, compensates for the occurred trajectory error. This way high speed smooth obstacle avoidance can be achieved, be it a full evasive manoeuvre or just a velocity modification.

As all the tools I have developed are general, they have much broader application possibilities.

1. *Multi-objective Genetic Algorithms with Quality-dominance and Measurement-based Ranking*

The proposed vector comparison operators are strict partial order binary endo-relations, being irreflexive, antisymmetric and transitive – thus they are uniformly usable in any mathematical or engineering process where a decision is to be made based on multipole criteria. The proposed metrics, including those for Pareto and weighted sum operators can be the bases for any ranking process, not just stochastic search, evolutionary algorithms and genetic algorithms. These proposed methods are computational efficient and provide detailed information on the quality, the nature and extent of difference between vectors of the same kind.

These new ranking and vector comparison methods can be freely use in any mathematical, engineering, economics or any other field, when objects of multiple properties are to be objectively compered or ranked. They are very much needed when the task is to optimise very complex, highly nonlinear system as fuzzy UAV flight dynamics models.

Based on the presented analysis my conclusion is that if one does not want to mess with vector comparisons, then the simple weighted sum of objectives will still do the trick; the only recommendation I give for this simple approach is to use the dominance approach to ranking (DO or DM) – measure by how much an individual is better than the others (and not by how much it is worse than the others), as this is a more efficient approach – observe the yellow highlighted D.DO.GA of Table IV.

Finally to offer an alternative to all those that still insist on using the classical Pareto vector comparison for multi-objective GAs: please observe the orange marked P.DM and P.DO GAs in Table IV to conclude that it is still more efficient to base the rank of an individual on the number of how many individuals it dominates (dominance based ranking) instead of looking for how many individuals do not dominate it (non-dominance based ranking).

2. Free Parametrisation Method for Unconstrained Tuning of Zadeh-type Fuzzy Partitions

My proposal for a successful fuzzy identification strategy is to take the 'GAzFLS' method as an off-line preliminary identification method, apply the results while keeping a continuous real-time 'LinLSzFLS' update mechanism in place for continuous fine tuning with fresh measurements, thus ensuring adaptability of the system.

The proposed fuzzy partition representation method performs exceptionally well when it comes to precision and reduced complexity (simplicity) of the solution format – low number of MFs and fuzzy rules. The global search of nonlinear parameters can be performed in a satisfactory fast manner by a well-constructed GA; gradient descent methods can simply and efficiently fine-tuned the system. With all linear parameters being LS optimal, the final quality of the identification is very good. Since the FLS complexity is reduced (low number of MFs and fuzzy rules) the model evaluation is fast. After the necessary offline pre-processing to calculate the system ***a*** then ***b*** and ***c*** parameters, for each online input data we need to perform only 5*(number of system inputs) multiplications and maximum 7*(number of system inputs), in average 4*(number of system inputs) additions; this can all be performed online in real time on practically any simple processor.

The number of multiplications and additions comes from these considerations:

- one input triggers maximum 2 MFs in a fuzzy partition;
- 2 MFs of equation (25) take maximum 3 additions (subtractions) and 2 multiplications (1 multiplication and 1 division); notice that the MF denominator does not take a subtraction for every online input calculation, it is constant, thus it can be pre-calculated; and actually the average number of additions is only 2;
- for each input it takes 2 multiplications for each antecedent of equation (22);
- for each input it takes 1 multiplication and 1 addition for each fuzzy consequence evaluation (24);

It is interesting to note that the proposed antecedent structure has all the positive properties of a second order B-spline antecedent. It has the same minimal number of parameters. Its derivative is continuous up to the second order. The evaluation of the complete rule base for every input can be omitted the same way as for B-splines. Furthermore, the proposed formulation has other benefits over B-splines: no iterative evaluation of the MF values is required. The MF parameters can be directly tuned by gradient based methods. No constraints have to be taken into consideration for its parameters. The consistency of the linguistic values remains intact throughout any fine-tuning so incorporated human knowledge can be fine-tuned without loss of meaning. Instead of nonlinear LS the proposed formulation and unconstrained gradient based method can be used for tuning the consequent parameters of a Mamdani type FLS, too.

3. Grey-box Genetic Fuzzy System Modelling and Control of Complex Dynamic Systems

The proposed general identification method is capable of forming and fine-tuning a soft computing, fuzzy system based dynamic models for any mechanical system that can be described by Euler-Lagrange equations, including but not limited to robotic manipulators, cranes, even for flight dynamics. The structure of the model is such that we can guarantee continuity of the model output and also periodicity if required. The model continuity and its bounded nature can be used for mathematically proving stability of control systems using the model. Periodicity of the model is needed to ensure the proper natural behaviour after full circle turns in flight dynamics or with robotic arms where a joint (typically a wrist) can rotate in one direction more than 360 degrees.

This method can be freely used for large, complex systems of many heavily coupled variables, as the number of nonlinear parameters is kept to minimum, and influenced only by the inertia matrix size; the complexity of the Coriolis and the centrifugal components do not introduce any new variables, as they are fully defined by and calculated from inertia components.

The structure of the model is such that after an initial evolutionary nonlinear parameter optimisation, it is possible to continuously, real-time fine tune the linear parameters of the model thus resulting in an adaptive model.

4. Feasible Optimal Harmonic Trajectories of Bounded, Smooth Time Derivatives

The proposed trajectory design method results in real-life feasible smooth, bounded torque transients, which is energy efficient for control signal design; while providing a flexible interface to arbitrary velocity, acceleration, jerk and snap limit enforcement. For such cases the trajectory is designed in a way to hold the maximal velocity, acceleration, jerk and snap, so that the desired lesser derivative maximum values are reached without any increase in higher derivatives. Figure 59 presents such trajectory, where the maximum snap is held for 1 second (yellow vertical lines), maximum jerk for 2 seconds (orange double vertical line), maximum acceleration for 4 seconds (pink rectangles) and the maximum velocity for 8 seconds (hollow red rectangles).

Reduction of this method is strait forward for more simple systems where it is enough to have smooth trajectories up to the 3rd time derivative of displacement. The notion of time and energy optimality is not used in some mathematics theory manner but in real life physically feasible engineering manner.

The same basic principle of accounting for system oscillations and the actuator dynamics when planning for system trajectories can be also applied to a crane model and any other than electro motor actuated system, by replacing equations (55) to appropriate ones, and then evaluating their transient behaviour. When the actuator dynamics and its relation to the system trajectory is known, one can use the algorithm and the method described in this paper to design trajectories of required transient dynamics and smoothness by replacing equation (62) to the appropriate one [s6].

These real time generated trajectories can be applied as parametrisation to any vector function defined path $f(s)=(x(s),y(s),z(s))$, which is chosen to accomplish the desired RM or crane task; also for complex UAV missions of arbitrary designed paths, even those including sudden unplanned changes as in obstacle avoidance. When determining the constraints on trajectory derivatives, one has to take into consideration both the system limits (39) or (43) and curvature properties of $f(s)$. The capability of real time trajectory generation with arbitrary constraints on displacement derivatives makes this trajectory design method especially suited for tactical re-planning of flight trajectories.

This paper presents results with feed-forward control scheme, but the trajectory design is applicable to any scheme including feedback setups. Naturally the poles of the system will change in a feedback loop, this will result in changed system dynamics – the period of a critical aperiodic transient will be different. This transient period can either be calculated as in equation (63) or simply measured as in equation (62) and then used as the parameter P in equations (64) and (65).

5. Singular Value Decomposition Based Genetic Fuzzy System Training Data Set Reduction

The proposed training data set reduction method is a general procedure usable for all identification procedures where a significant part of the approximation is linear – as in linear parameter approximation in TSK fuzzy system consequent part. For complex system identification problems we must be sure that the training data set is sufficiently exciting – that it reveals all the typical modes of the system. Datasets of sufficiently exciting system trajectories tend to be oversized. Oversized data is expensive to evaluate, especially in iterative and evolutionary search methods.

The need to reduce the training dataset is real, but for many applications the quality of the result is of outmost importance as in model based control of UAV flight dynamics we must not allow for any unknown, uncontrolled states. Using this proposed training data reduction method we can ensure that we have full control over the quality and also the size of the training data, thus full control off the uniform quality of our UAV flight dynamics model and implicitly of the stability of the model based flight control.

REFERENCES

[12] J. Coelho, R. Neto, C. Lebres, V. Santos, “Application of Fractional Algorithms in Control of a Quad Rotor Flight,” Proc. 2nd Conference on Nonlinear Scienceand Complexity, Porto, Portugal, July 28-31, 2008, pp. 1-12.

[23] C. M. Fonseca, P. J. Fleming, “Multiobjective Optimisation and Multiple Constraint Handling with Evolutionary Algorithms I: A Unified Formulation,” Technical Report 564, Univ. of Sheffield, Sheffield, UK, 1995.

[33] C. Lebres, V. Santos, N. M. Fonseca Ferreira, J. A. Tenreiro Machado, “Application of Fractional Controllers for Quad Rotor,” Nonlinear Science and Complexity, Part 6, Springer, 2011, pp. 303-309.

[35] R. Lozano, “Unmanned Aerial Vehicles”, ISTE Ltd, London, 2010.

[37] D. Mellinger, V. Kumar, “Minimum snap trajectory generation and control for quadrotors,” IEEE Int. Conf. on Robotics and Automation (ICRA2011), 2011, pp. 2520 –2525.

[57] W. H. Press, et.all, “Numerical Recipes in C, The Art of Scientific Computing,” Cambridge University Press, Cambridge, 1990.

[60] A. Rodić, G. Mester, “Modeling and Simulation of Quad-Rotor Dynamics and Spatial Navigation,” Proc. 9th IEEE Int. Symp. on Intelligent Systems and Informatics SISY 2011, Subotica, Serbia, 8–10 September, 2011, pp 23-28.

[63] A. Rodić, G. Mester, I. Stojković, "Qualitative Evaluation of Flight Controller Performances for Autonomous Quadrotors," in E. Pap (Ed.): Intelligent Systems: Models and Applications, Springer-Verlag Berlin Heidelberg, 2013, Vol.3, pp. 115–134.

[80] The ZODIAC, "Theory of Robot Control," Springer-Verlag London Ltd., 1996.

Publications in Support of Thesis

[s1] A. Nemes, "Synopsis of Soft Computing Techniques Used in Quadrotor UAV Modelling and Control," Interdisciplinary Description of Complex Systems, 2015, Vol.13, No.1, pp. 15-25.

[s2] A. Nemes, "New Genetic Algorithms for Multi-objective Optimisation," Proc. 1st Int. Symp. of Hungarian Researchers on Computational Intelligence (HUCI 2000), Budapest, 2000.

[s3] A. Nemes, "Function Identification by Unconstrained Tuning of Zadeh-type Fuzzy Partitions," Proc. 2nd Int. Symp. of Hungarian Researchers on Computational Intelligence (HUCI 2001), Budapest, 2001.

[s4] A. Nemes, "System Identification Based on Multi-Objective Optimisation and Unconstrained Tuning of Zadeh-type Fuzzy Partitions," Proc. 2nd IEEE Int. Symp. Intelligent Systems and Informatics (SISY 2003), Subotica, Serbia, 2003.

[s5] A. Nemes, "Dynamic Modelling of Robot Manipulators by Zadeh-type Fuzzy Partitions," Proc. 4th Int. Symp. Hungarian Researchers on Computational Intelligence (HUCI 2003), Budapest, 2003.

[s6] A. Nemes, "Continuous Periodic Fuzzy Logic Systems and Smooth Trajectory Planning for Multi-Rotor Dynamic Modeling," Acta Polytechncia Hungarica, 2016, Vol.13, No.6, pp. 215-234.

[s7] A. Nemes, "Fuzzy-Genetic Control of Quadrotors Unmanned Aerial Vehicles," Interdisciplinary Description of Complex Systems, 2016, Vol.14, No.2, pp. 223-235.

[s8] A. Nemes, "Genetic Algorithm-Based Adaptive Fuzzy Logic Systems for Dynamic Modeling of Quadrotors," Proc. 3rd Int. Conf. MechEdu, 2015, 14-15 May, Subotica, Serbia, pp. 96-103.

[s9] A. Nemes, "Genetic Fuzzy Identification Method for Quadrotor UAVs," Annals of Faculty of Hunedoara – International Journal of Engineering, 2015, Tome XIII, Fascicule 3, pp. 257-264.

[s10] A. Nemes, B. Lantos, "Genetic Algorithms-Based Fuzzy Logic Systems for Dynamic Modeling of Robots," Periodica Polytechnica Electrical Engineering, 1999, Vol.43, No.3, pp. 177-187.

[s11] A. Nemes, B. Lantos, "Optimization of Fuzzy Logic Systems for Gray-Box Dynamic Modeling of Robot Manipulators by Genetic Algorithms," Proc. IEEE Int. Conf. on Intelligent Engineering Systems INES'99, 1999. pp. 353-358.

[s12] A. Nemes, B. Lantos, "Training Data Reduction for Optimisation of Fuzzy Logic Systems for Dynamic Modelling of Robot Manipulators by Genetic Algorithms," Proc. IEEE Instrumentation and Measurement Technology Conf. (IMTC 2001), 2001, Vol.3, pp. 1418-1423.

[s13] A. Nemes, G. Mester, “Unconstrained Evolutionary and Gradient Descent-Based Tuning of Fuzzy partitions for UAV Dynamic Modeling,” FME Trans., 2017, Vol.45, No.1, pp.1-8

[s14] A. Nemes, G. Mester, “Energy Efficient Feasible Autonomous Multi-Rotor Unmanned Aerial Vehicles Trajectories,” Proc. 4th Int. Scientific Conf. on Advances in Mechanical Engineering (ISCAME), 2016, Vol.1, pp. 369-377.

[s15] A. Nemes, “Feasible Time and Energy Optimal, Minimum Oscillations Trajectory Design,” Proc. XXXII Kandó Conf. (KSC), 2016

[s16] A. Nemes, “Robotkarok leegyszerűsitett szürkedoboz identifikációja fuzzy függvényekkel,” Proc. MicorCAD 2001, Miskolc, Hungary, 2001.

Further Publications

[87] G. Mester, A. Nemes, S. Pletl, T. Mester, “Soft Computimg metoda za optimalno modeliranje buke,” Proc. YOUINFO’99, Kopaonik, Yugoslavia, 1999.

[88] G. Mester, S. Pletl, A. Nemes, T. Mester, “Structure Optimization of Fuzzy Control Systems by Multi-Population Genetic Algorithm,” Proc. 6th European Congress on Intelligent Techniques and Soft Computing, EUFIT’98, Verlag Mainz, Aachen, Germany, 7-10. September 1998, Vol.1, pp. 450–456.

[89] G. Mester, A. Nemes, S. Pletl, J. Varga, “Optimization of Electric Motors by Multi-Population Genetic Algorithm,” Proc. SYMOPIS, Zbornik radova Jugoslovenskog simpozijuma o operacionim istrazivanjima, Herceg-Novi, Yugoslavia, 1998, pp. 8-9.

[90] G. Mester, A. Nemes, P. Odri, S. Pletl, “Design of the Optimal Fuzzy Modeling of Noise Pollution Using Multi-population Genetic Algorithm,” Proc. SCPE’97, Nis, Yugoslavia, 1997.

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